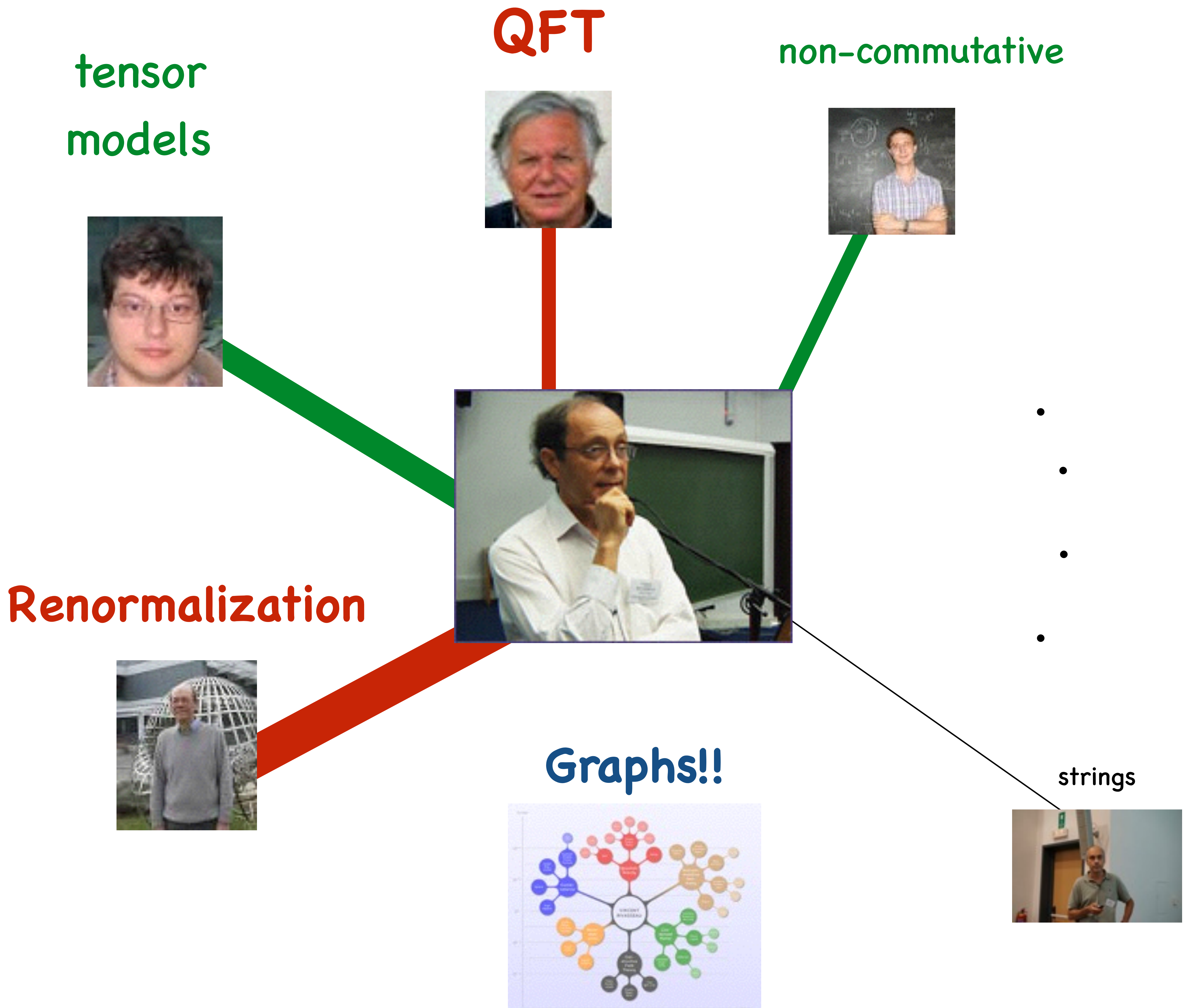
Vincent is 60 !

C. Bachas, ENS - Paris

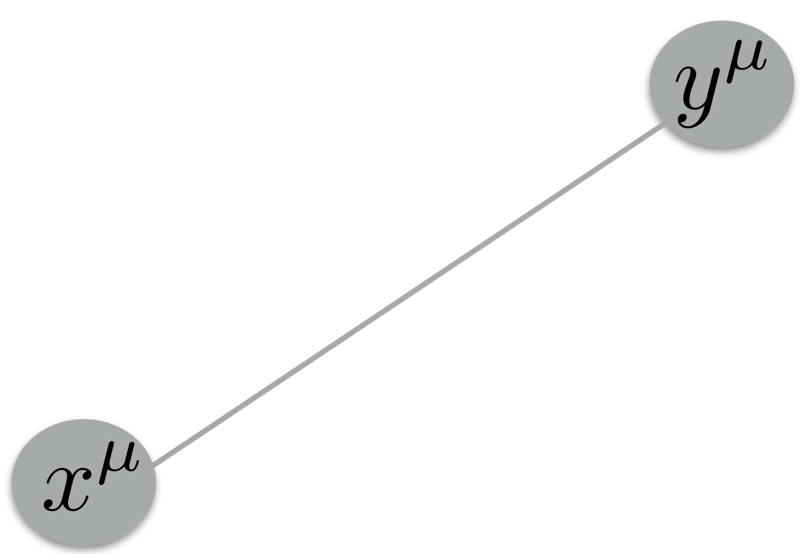
IHP, November 2015


QFT
tensor
models
non-commutative
Renormalization
Graphs!!
strings

Quantum Field Theory

is defined by local operators (fields) and their correlation (or Green, or Wightman) functions

Local fields perturb the vacuum at (smeared) points in space and time, so their Green fncts enjoy analyticity properties due to causality



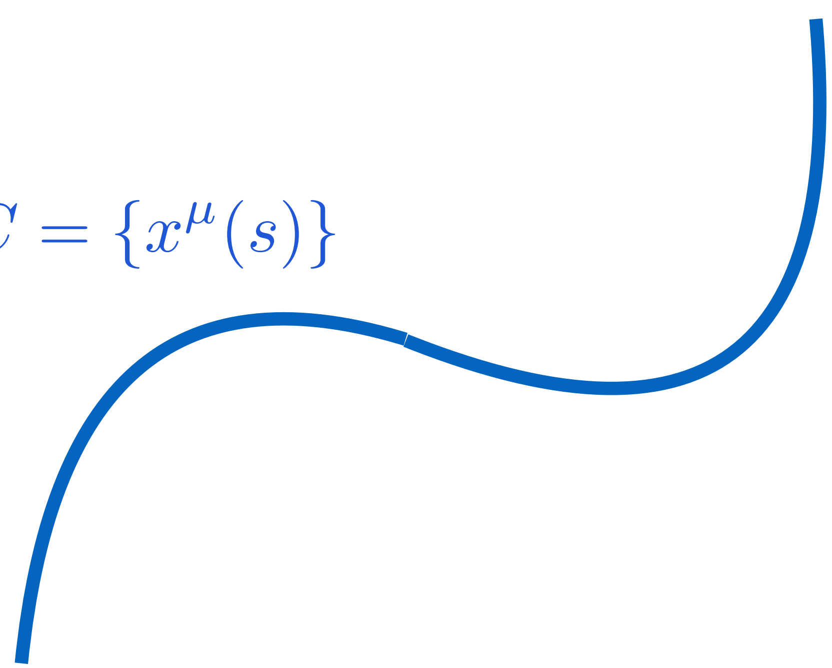
$$\langle \phi(x)\phi(y)\rangle = \frac{\delta}{\delta j(x)}\frac{\delta}{\delta j(y)} Z(j)$$

But all QFTs admit rich sets of **non-local observables:**

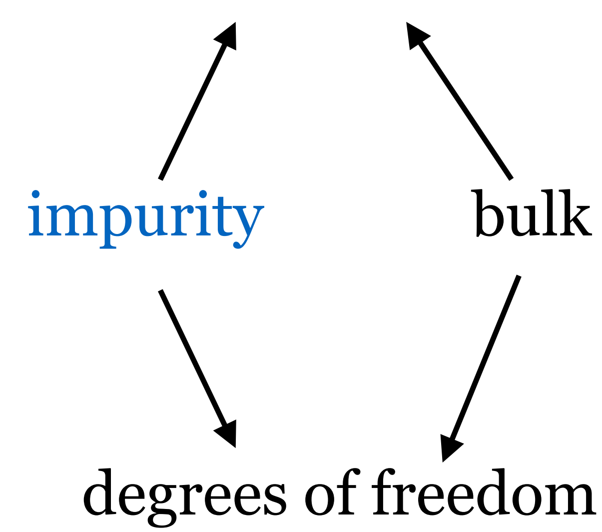
→ **Line operators** : worldlines of quantum impurities

$$\langle W(C)\rangle = \langle\, \mathrm{tr}(T e^{i\int_C H_{\rm imp}})\,\rangle$$

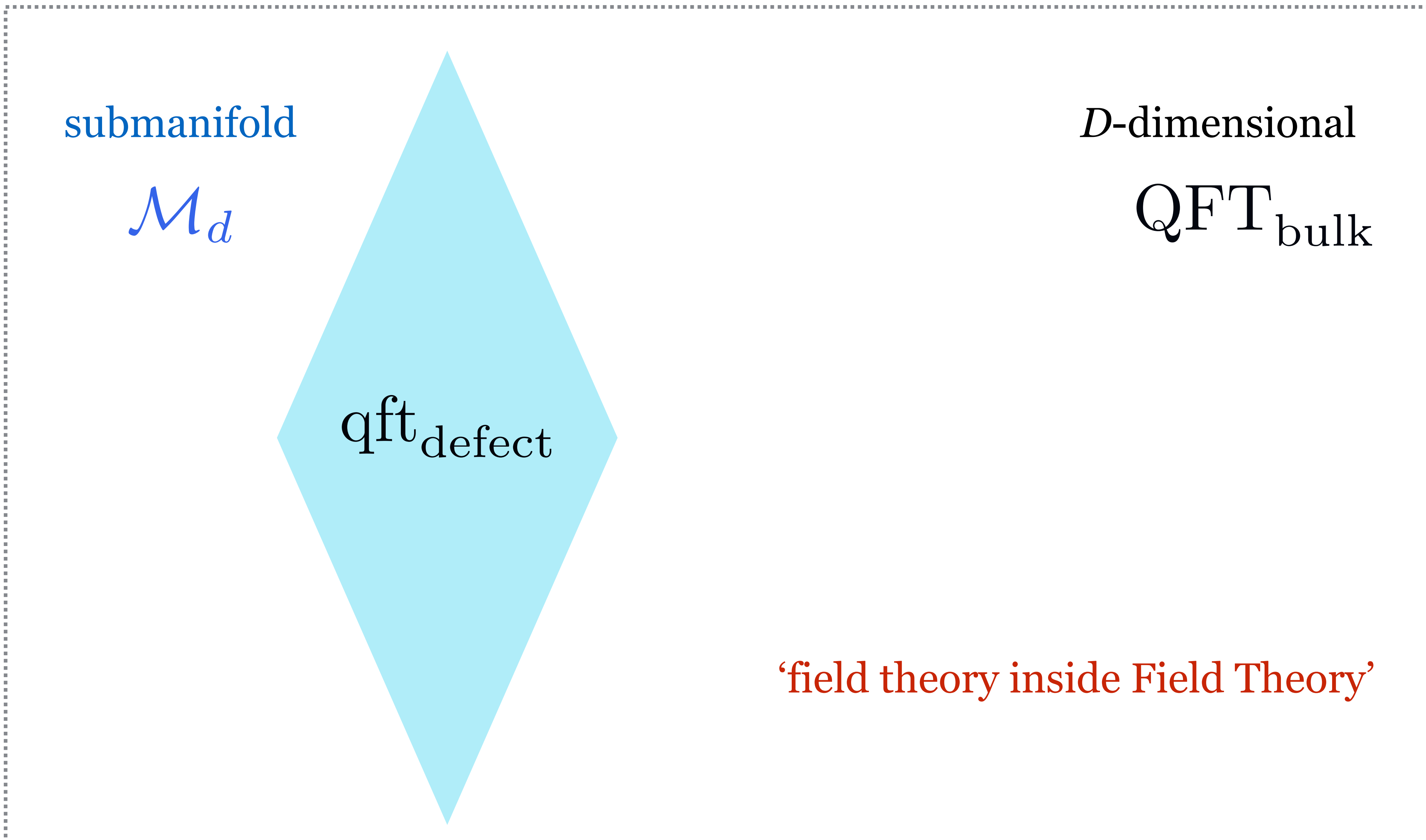
$C = \{x^\mu(s)\}$



$$H_{\rm imp}(\chi(s), \phi(x(s)))$$



Surface, volume etc operators



Bulk fields are **couplings** in $\mathrm{qft}_{\mathrm{defect}}$ so $Z(\mathrm{qft}_{\mathrm{defect}})$

is a non-local operator of $\mathrm{QFT}_{\mathrm{bulk}}$

'Quantum Defects' uncover QFT properties that cannot be probed by local operators alone:

→ **Kondo** impurity (quantum dots)

birth of the Renormalization Group

→ **Wilson** loop in gauge theories

order parameter for confinement

They offer a (largely unexplored) Pandora's box for aficionados of QFT, like Vincent !

Fleury of activity in past few years, with varying motivations:

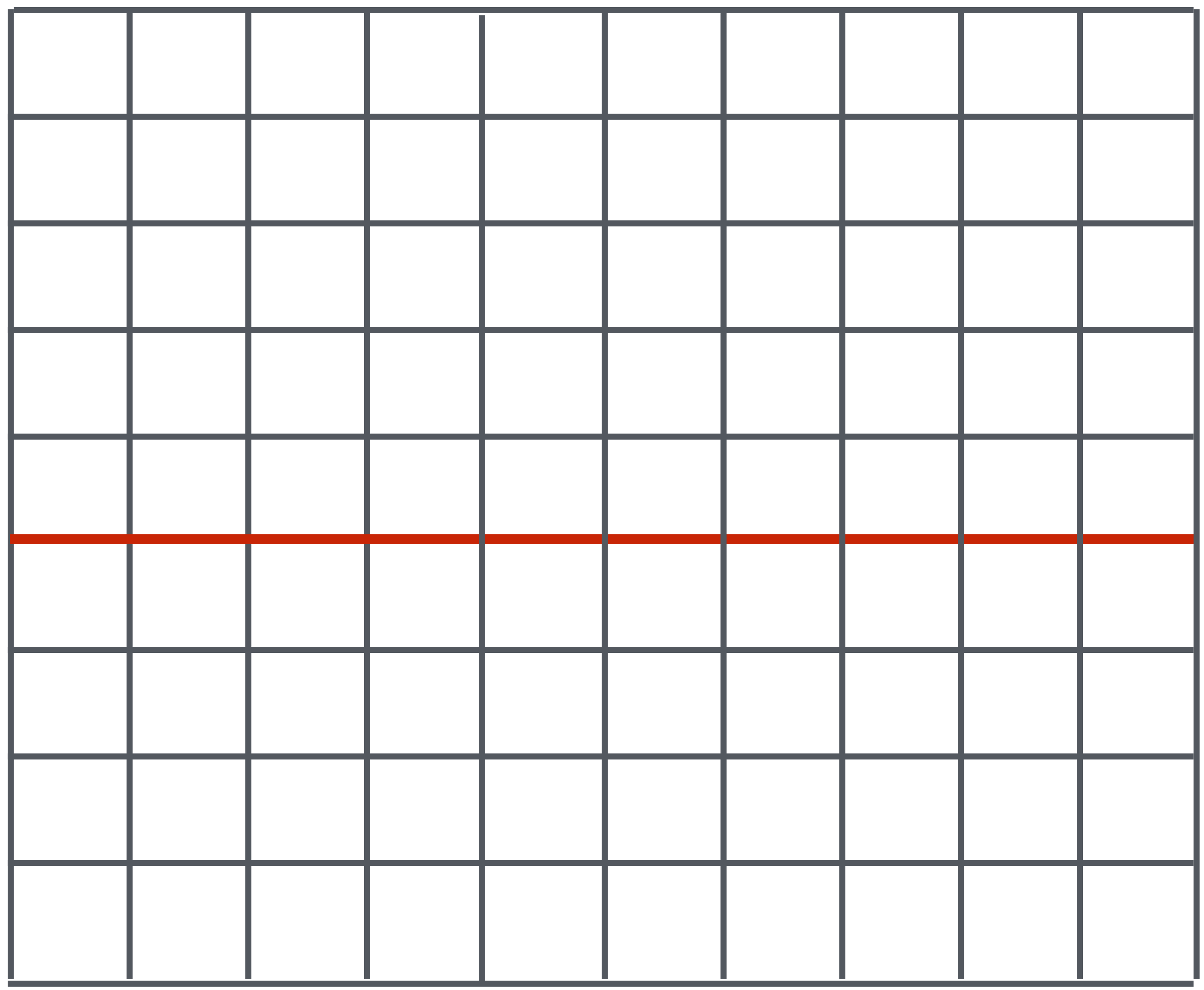
Condensed matter, D-branes & dualities, Langlands program, localization of path integral, gravity localization, ...

I restrict myself in the remaining time to 2 simple examples that give a flavor of recent results and open problems

1 Line defects in the 2d Ising Model

(need math foundation; resurgence ?)

$$Z = \sum_{\sigma_j = \pm 1} e^{-\sum_{<ij>} J_{ij}\sigma_i\sigma_j}$$



$J_{ij} = J_2$ (vertical bonds)

$J_{ij} = J_1$ (horizontal bonds)

defect coupling

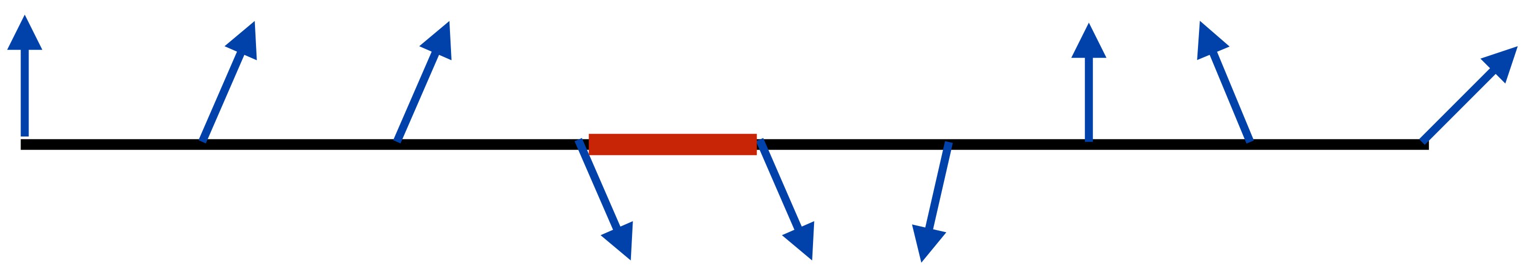
$J_{ij} = bJ_1$

Critical couplings

$$\sinh(2J_1^\star)\sinh(2J_2^\star) = 1$$

2D Ising model = 1D quantum spin chain

in the limit $J_2 \to \infty$



$$H_D = -\sum_n \sigma_n^x - \sum_{n\neq 0} \sigma_n^z \sigma_{n+1}^z - b\ \sigma_0^z \sigma_1^z$$

defective coupling

$$H_N = -\sum_n \sigma_n^x - \sum_{n\neq 0} \sigma_n^z \sigma_{n+1}^z - \tilde{b}\ \sigma_0^z \sigma_1^x$$

[twisted]

Questions:

- Are the parameters $b, \tilde{b}$ renormalized ?

Answer: No, they are marginal couplings

Henkel *et al* ‘89
Abraham *et al* ‘89
Oshikawa, Affleck ‘96

- Does the critical Ising model have any other defect lines ?

Answer: most likely not (but no proof)

Friedan, *unpublished* ‘00

Gaberdiel, Recknagel ‘01

- What is the **fusion** of two defect lines?

Answer: a deformation of the group algebra of $O(1,1)/\mathbb{Z}_2$

CB, Brunner, Roggenkamp ‘13

Fusion

[analog of **Operator Product Expansion** for local operators ?]

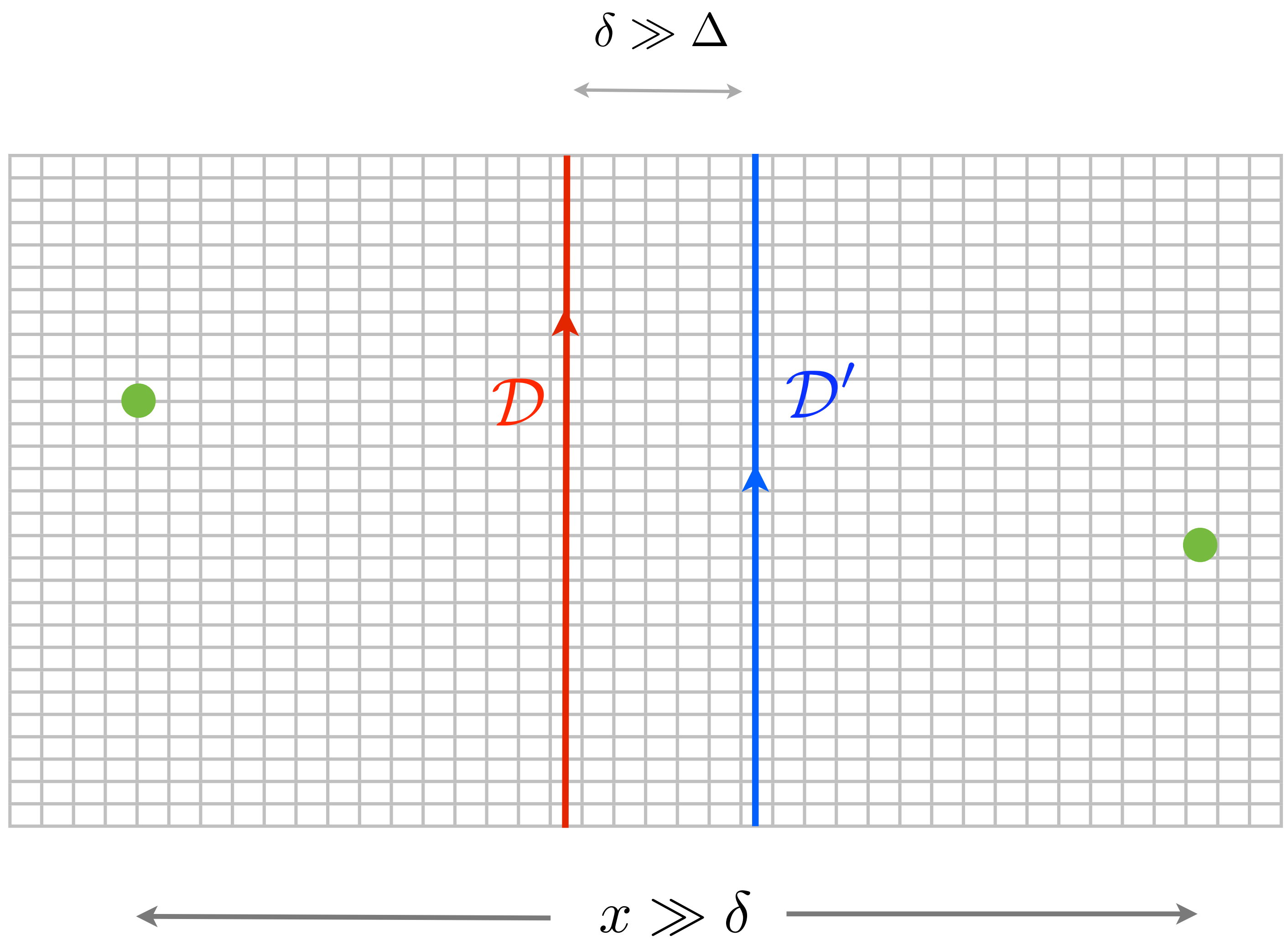
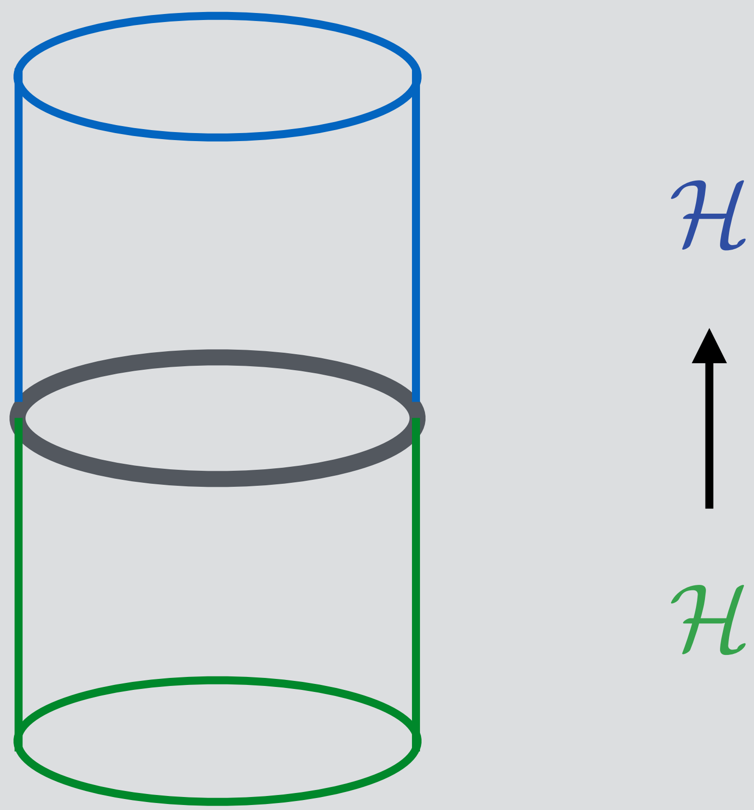


Figure 1: The two-defect system discussed in the text. The green dots are arguments of a typical two-point function at a horizontal scale $x \gg \delta$. The fusion product $\mathcal{D} \star \mathcal{D}'$ gives an effective description of this system in the limit where δ is very large compared to the lattice spacing Δ. Only in this limit is fusion universal.

The 2D Ising model is **integrable**, and the critical model is a **free** (Majorana fermion) field theory

But these results were obtained by (algebraic) techniques of **CFT :**

Think of defect lines as **formal operators** acting on states on S^1



$$\mathcal{D} = g|0\rangle\langle 0| + \cdots$$

$\log g =$ entropy

They can be added or multiplied (?) $\Longrightarrow$ They form an **algebra**

To construct $\mathcal{D}$ use techniques of **Boundary Conformal Field Theory**

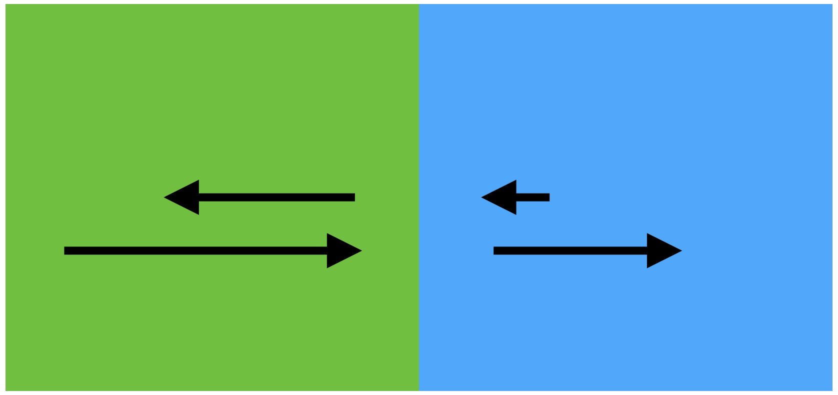
Cardy '89

....

CB, de Boer, Dijkgraaf, Ooguri '01

A conformal defect line (interface) cann't trap energy, so the energy-flux across it must be continuous:

$$T_{xt} = T_{xt} \quad \Longrightarrow \quad [\mathcal{D}, L_n - \bar{L}_{-n}] = 0$$



special case:

$$[\mathcal{D}, L_n] = [\mathcal{D}, \bar{L}_n] = 0$$

`topological'

Petkova, Zuber '00

3 one-parameter families of conformal defects :

ferromagnetic	$b \in [0, \infty]$	$\mathbb{I},\ \Lambda = \begin{pmatrix} \cosh\gamma & \sinh\gamma \\ \sinh\gamma & \cosh\gamma \end{pmatrix}$
antiferromagnetic	$b \in [-\infty, 0]$	$\epsilon,\ \Lambda = \begin{pmatrix} \cosh\gamma & \sinh\gamma \\ \sinh\gamma & \cosh\gamma \end{pmatrix}$
order-disorder	$\tilde{b} \in [0, \infty]$	$\sigma,\ \Lambda = \begin{pmatrix} \cosh\tilde{\gamma} & -\sinh\tilde{\gamma} \\ \sinh\tilde{\gamma} & -\cosh\tilde{\gamma} \end{pmatrix}$

square lattice: $\gamma = \log \left| \frac{\tanh(bJ_1^\star)}{\tanh(J_1^\star)} \right|$

quantum chain: $e^{\gamma} = |b|, \quad e^{\tilde{\gamma}} = \tilde{b}$

universal parametrization

$\Lambda \in O(1,1)/\mathbb{Z}_2$

gluing $\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

Fusion algebra : $\mathcal{D}_I(0)\,\mathcal{D}_J(\delta) \sim \sum c_{IJ}^{\;\;K}\,\mathcal{D}_K(0)$

$\simeq e^{-A/\delta}$
drops out in corln functions

$$(a,\Lambda)\odot(a',\Lambda') = (a\times a', \Lambda\Lambda')$$

for **2D Ising :**

$$\epsilon\times\epsilon = \mathbb{I}$$
$$\epsilon\times\sigma = \sigma$$
$$\sigma\times\sigma = \mathbb{I}+\epsilon$$

Verlinde algebra

NB: The defects with $|b|, \tilde{b} = 1$ are topological defects that implement the reflection and **Kramers-Wannier** duality of the model

Froehlich, Fuchs, Runkel, Scheigert '04

2 Wilson loops in N=4 SYM

(geometry of resummed Feynman graphs)

$\mathcal{N}=4$ SYM is the dimensional reduction of 10d SYM:

$$\mathcal{L} = -\frac{1}{2g^2}\mathrm{tr}(F_{MN}F^{MN}) - \bar{\lambda}(i\partial_M - A_M)\gamma^M\lambda$$

$$M \in \{0,1,2,3,\ 4,5\cdots 9\}$$

$$A_\mu \qquad \Phi_a \longleftarrow$$ adjoint of SU(n)

parameters: $n,\ \ \lambda = g^2 n$

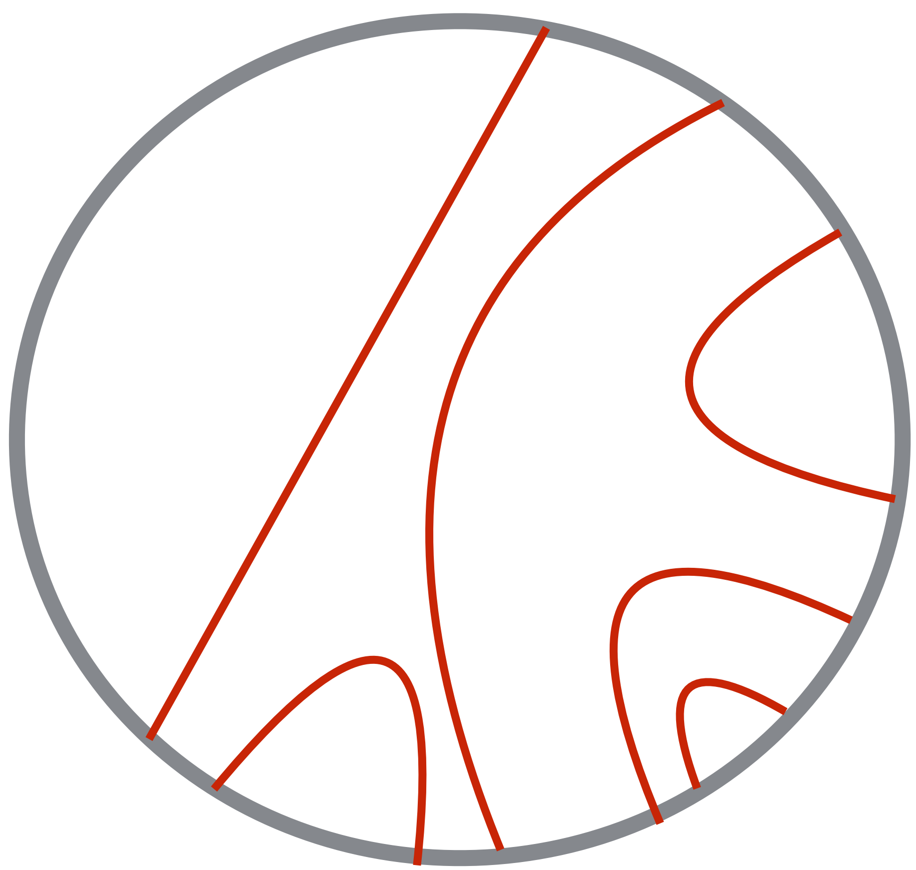
Wilson - Maldacena loop: $$Pe^{i\oint(A_\mu \dot{x}^\mu + \kappa\hat{n}\cdot\vec{\Phi})ds}$$

κ runs, $\kappa=1$ stable fixed point

For a circular loop at $\kappa = 1$ the answer can be computed exactly by localization, and the result is a Gaussian matrix integral:

$$\langle W_R(\text{circle})\rangle = \frac{\int [\mathcal{D}a]\, e^{-8\pi^2\langle a,a\rangle/g^2_{\rm YM}}\, \mathrm{Tr}_R e^{2\pi i a}}{\int [\mathcal{D}a]\, e^{-8\pi^2\langle a,a\rangle/g^2_{\rm YM}}}$$

Pestun '07



This resums **'rainbow diagrams'** which turn out to be the only that contribute *(interior vertices cancel)*

Erickson, Semenoff, Zarembo '00

Drukker, Gross '00

How about arbitrary (smooth) loops ?

Geometric limit for $\lambda, n \to \infty$?

AdS/CFT + D-brane `engineering'

Defect action: $$\int dt\, \mathcal{L}^{\rm D5}_{\rm defect} = \int dt \left[\chi^\dagger (i \frac{d}{dt} - A_0 - \Phi)\chi + k\, {\rm tr} A_0 \right]$$

1D fermion in fundmtal

$\Longrightarrow$ $H^{\rm D5}_{\rm defect} = \chi^\dagger (A_0 + \Phi)\chi$ **with projection on** $\chi^\dagger \chi = k$

$\Longrightarrow$ $T e^{i \int \mathcal{H}_{\rm defect} dt} = P e^{i \int (A_0 + \Phi) dt}$ in rank-k antisymmetric rep

This is a field theory corresponding to a **D5-brane** in **AdS5 x S5**

$$ds^2 = \frac{L^2}{y^2}\left(dy^2 - dt^2 + dr^2 + r^2 d\Omega_2^2\right) + L^2(d\psi^2 + \sin^2\psi\, d\Omega_4^2)$$

$$C_4 = 4L^4\left[\left(\frac{1}{32}\sin 4\psi - \frac{1}{4}\sin 2\psi + \frac{3}{8}\psi\right)\omega_4 + \frac{r^3}{3}\frac{dy}{y^5}\wedge dt\wedge\omega_2\right]$$

$$\Longrightarrow \qquad F_5 = 4L^4\omega_5 + *$$

$$S_{\rm D5} \simeq S_{\rm DBI} + S_{\rm WZ} + S_{\rm bndry}$$

$$S_{\rm DBI} = T_5\int d^6\sigma\sqrt{det(\hat{g}_{ab} + F_{ab})}$$

$$S_{\rm WZ} = T_5\int F\wedge\hat{C}$$

The Wilson loop for $\lambda, n \to \infty$ is the minimal $S_{\rm D5}$ for a 5-brane intersecting the AdS boundary on $S^4\times S^1$

The result: $$\langle W \rangle \simeq e^{\sqrt{\lambda}\,\frac{2n}{3\pi}\,\sin^3\psi_k}$$

where $$\frac{k\pi}{n} = \psi_k - \cos\psi_k \sin\psi_k$$

in perfect agreement with matrix model

Yamaguchi '06

Gomis, Passerini '06

How to see the emergence of the geometry from Feynman graphs ?

Other representations, non-circular loops, $\kappa \neq 1$?

....

in progress with M. Jarvinen

Joyeux Anniversaire Vincent !