A multi-scale tsunami:

Vincent's impact on group field theory renormalisation (and more)

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Max Planck Institute for Gravitational Physics
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Conference on "Constructive Field Theory",
in honour of Vincent Rivasseau's 60th Birthday
Paris, France, EU - 27/11/2015

early 2009 - about the time I first met Vincent….

the field of quantum gravity was developing, with lots of activities and results, but quietly and peacefully….

…. so, it attracted Vincent’s attention and interest….

little did we know…..

….. of what was about to happen….

little did I know…..

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Before Vincent……

Loop quantum gravity and spin foam models

started as canonical quantization of continuum GR in connection/triad variables $(A_a^i \quad , \quad E_i^b = \frac{1}{\gamma}\sqrt{e}\, e_i^b)$

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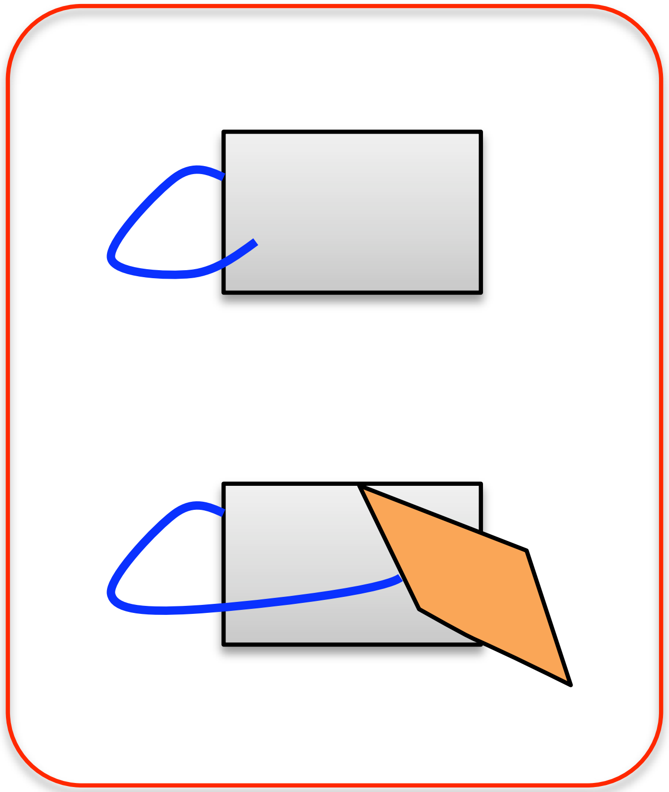
then, switch to “discrete counterparts: holonomies and fluxes:

$$h_e(A) = \mathcal{P}\, e^{\int_e A} \in SU(2) \quad \forall e : [0,1] \rightarrow \Sigma \qquad E_j(S) = \int_S (\star E_j)_{ab} dx^a \wedge dx^b \in \mathfrak{su}(2) \qquad \forall S \subset \Sigma$$



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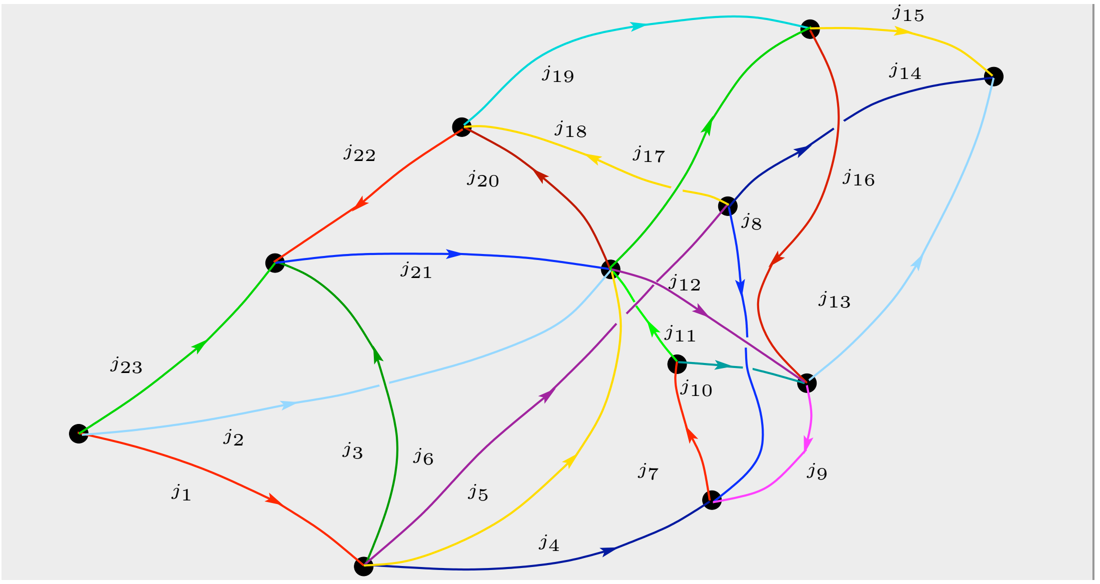
….. impose diffeo invariance………. end up with purely algebraic and combinatorial structures:

- Hilbert space decomposes into graph-based sectors: $\mathcal{H} = \bigoplus_{\Gamma} \mathcal{H}^{inv}_{\Gamma} \qquad \mathcal{H}^{inv}_{\Gamma} = L^2\left(SU(2)^E / SU(2)^V\right)$
- spin network representation (graphs labelled by algebraic data):

Rovelli-Smolin, '95

$$f^{inv}_{\Gamma}(g_1, ..., g_E) = \sum_{j_1,...,j_E} f^{j_1,..,j_E;i_v}_{\Gamma}\, s_{\Gamma;\, j_e\,,\, i_v}(g_e) \qquad s_{\Gamma;\, j_e\,,\, i_v}(g_e) = \prod_{v\in\Gamma} i_v(j_{e\supset v}) \prod_{e\in\Gamma} D^{j_e}(g_e)$$



intertwiner between representations on edges incident to vertex

Wigner representation function

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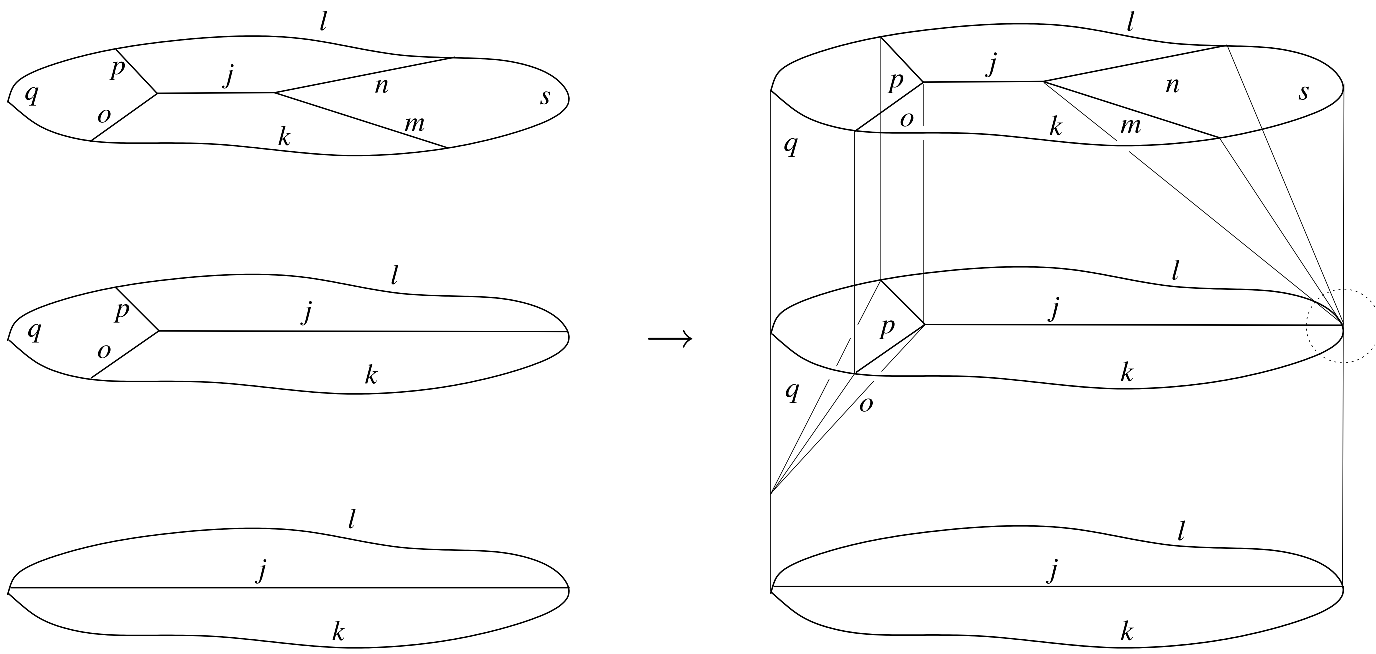
history is 2-complex (vertices, edges, faces) labelled by same algebraic data = spin foam



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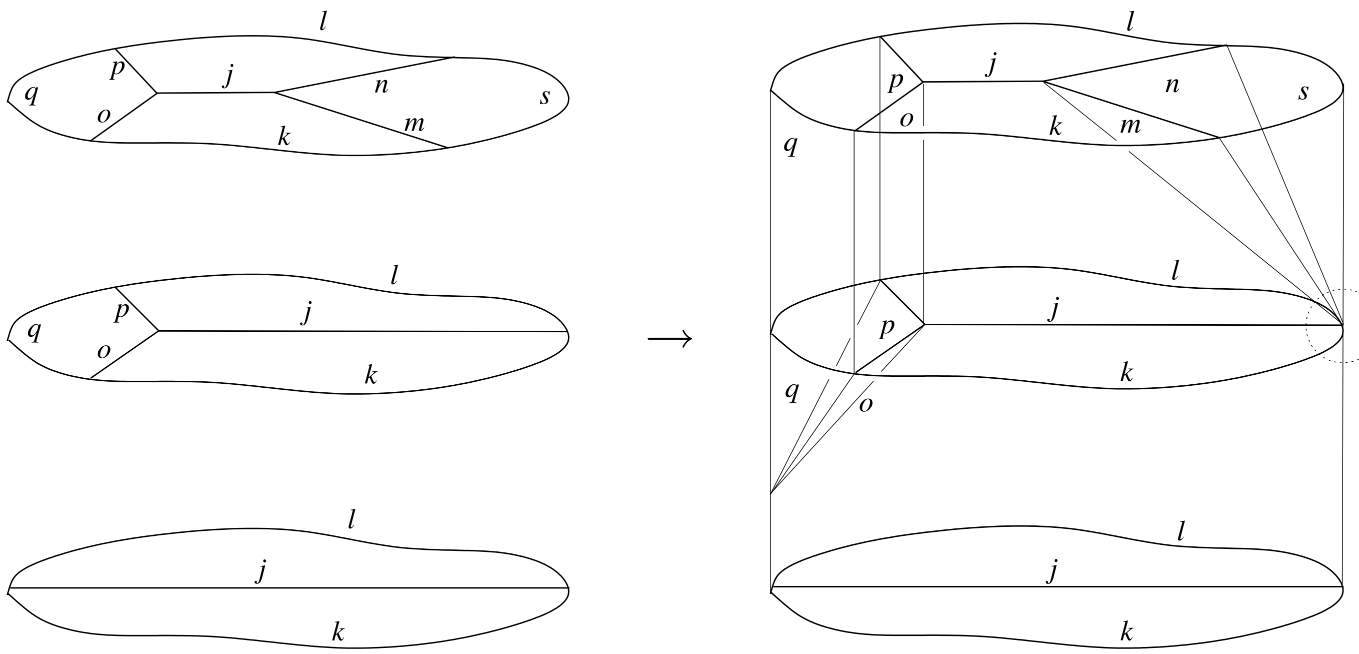

Transition amplitudes = sum over histories (spin foam model = combinatorial-algebraic sum over geometries):

$$\langle \Psi_\gamma(j,i)\,|\,\Psi_{\gamma'}(j',i')\rangle = \sum_{\Gamma|\gamma,\gamma'} w(\Gamma) \sum_{\{J\},\{I\}|j,j',i,i'} \mathcal{A}_\Gamma\,(J,I) \approx \text{“} \int \mathcal{D}g\, e^{i\,S(g)} \text{”}$$

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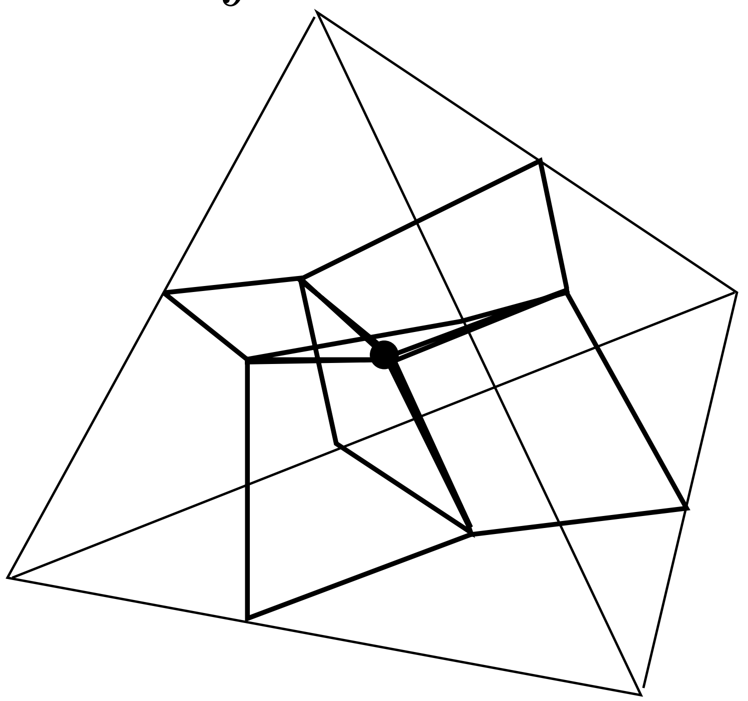

in fact,

- spin foam 2-complex dual to simplicial complex in d dimensions
- spin foam amplitudes (for given 2-complex) are simplicial gravity path integral (~ quantum Regge calculus) in different variables (group representations)
- sum over spin foam 2-complexes ~ sum over triangulations

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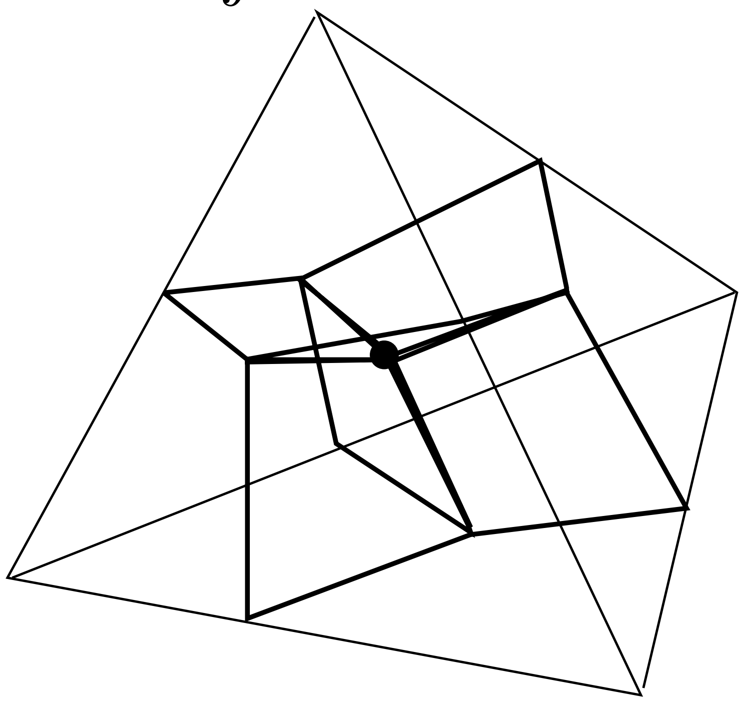

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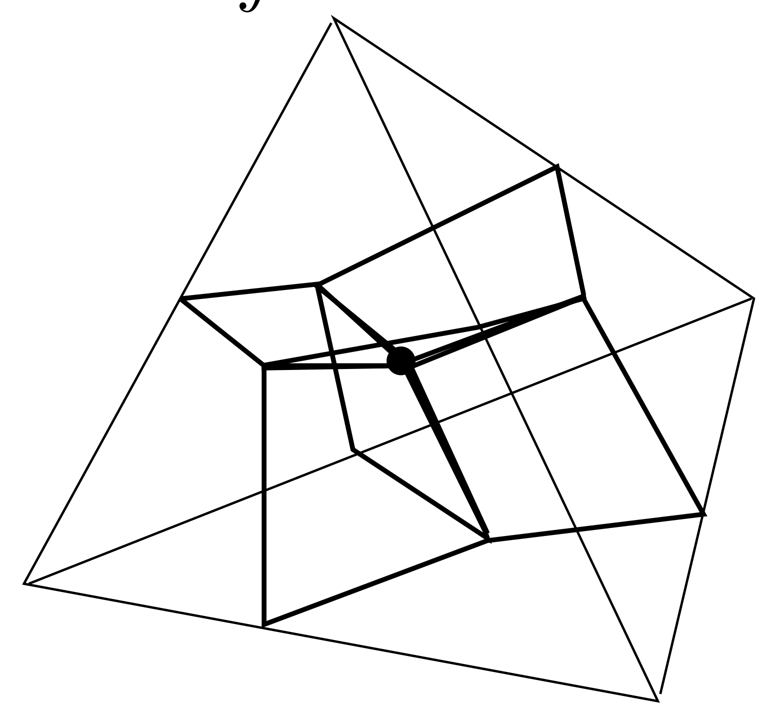

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many results in recent years:

- quantum geometric understanding of states and amplitudes
- several interesting models
- stronger link canonical <— —> covariant formalisms
- deeper link with simplicial gravity

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but….. how to define renormalisation for background independent quantum gravity,

i.e. for pre-geometric degrees of freedom, in absence of space and time?

Group field theories

(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, ……)

Quantum field theories over group manifold G (or corresponding Lie algebra) $\varphi : G^{\times d} \rightarrow \mathbb{C}$

QFT of spacetime, not defined on spacetime

relevant classical phase space for "GFT quanta": $\left(\mathcal{T}^* G\right)^{\times d} \simeq \left(\mathfrak{g} \times G\right)^{\times d}$

can reduce to subspaces in specific models depending on conditions on the field

d is dimension of "spacetime-to-be"; for gravity models, G = local gauge group of gravity (e.g. Lorentz group)

example: d=4 $\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \rightarrow \mathbb{C}$

can be defined for any (Lie) group and dimension d, any signature, ...

very general framework; interest rests on specific models/use
(most interesting QG models are for Lorentz group in 4d)

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Fock vacuum: “no-space” (“emptiest”) state $| 0 >$

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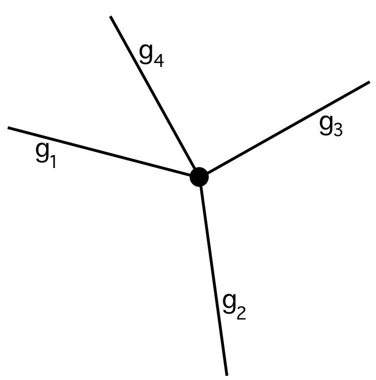

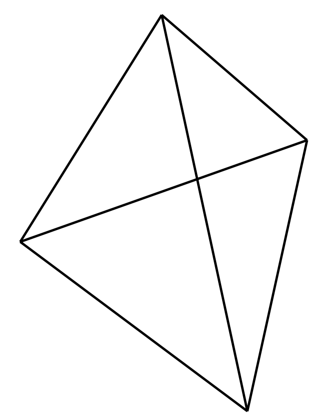

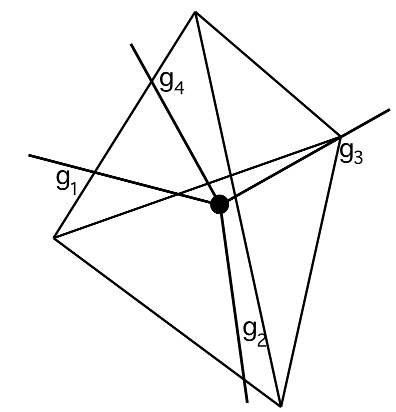

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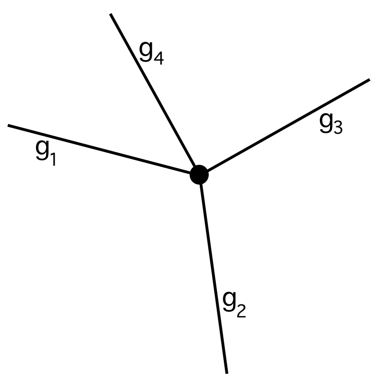

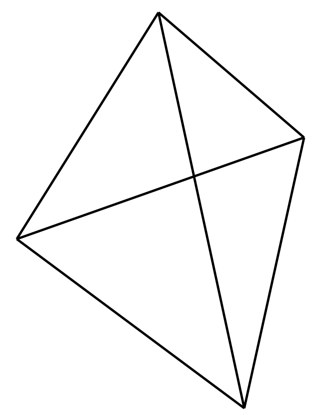

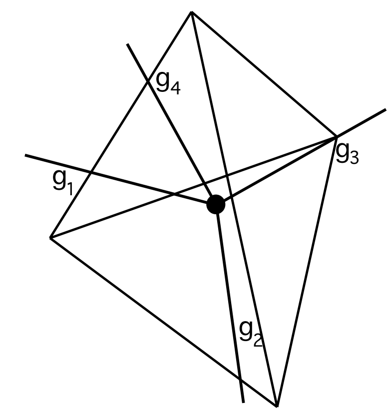

generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones) - same type of states as in LQG

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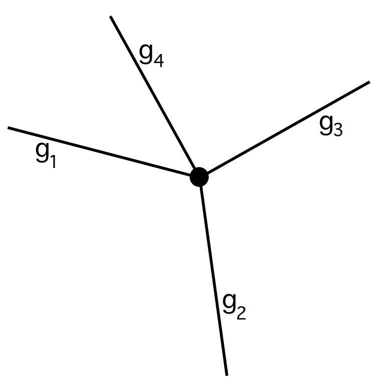

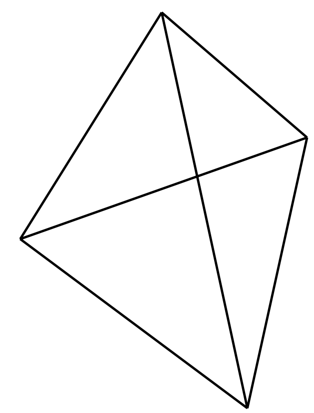

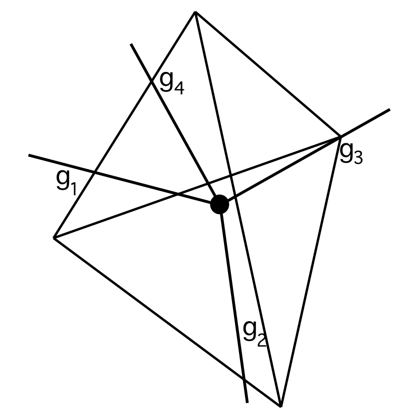

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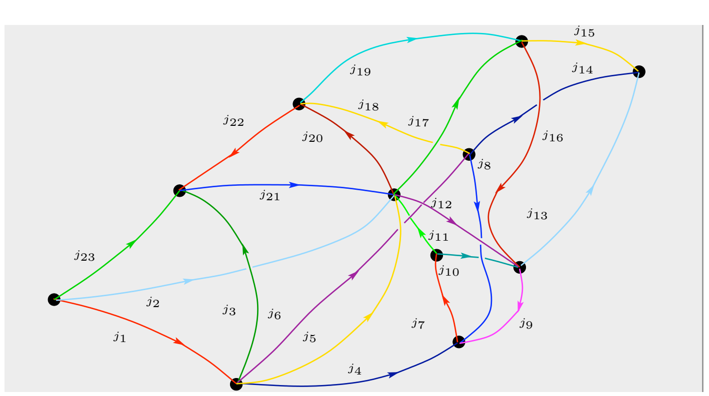

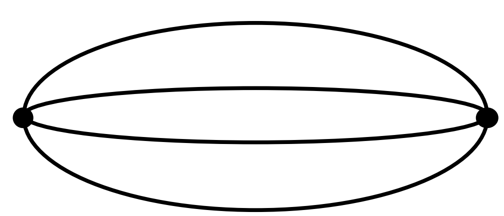

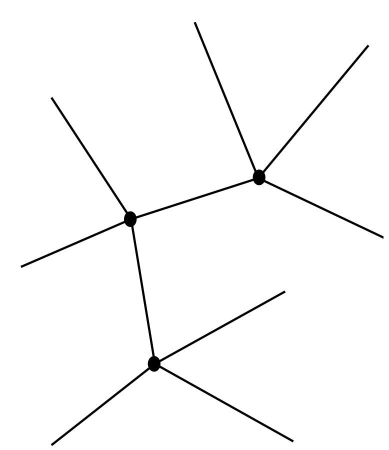

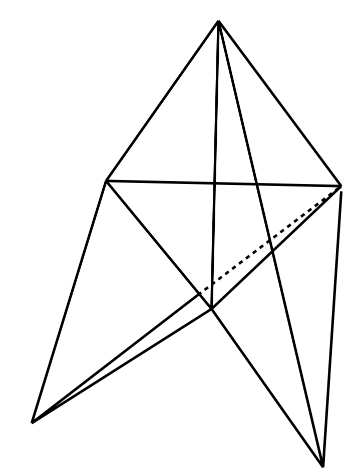

Group field theories

classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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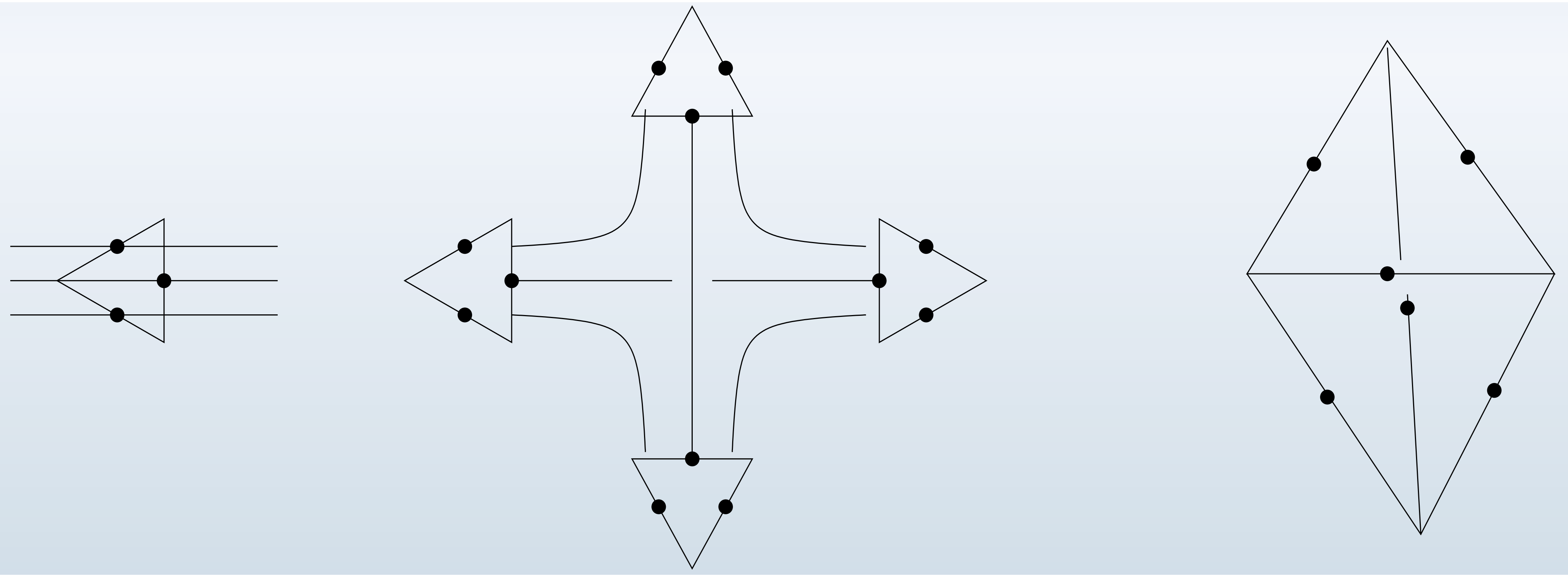

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combinatorics of field arguments in interaction: gluing of 5 tetrahedra across common triangles, to form 4-simplex ("building block of spacetime")

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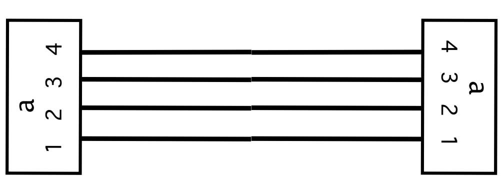

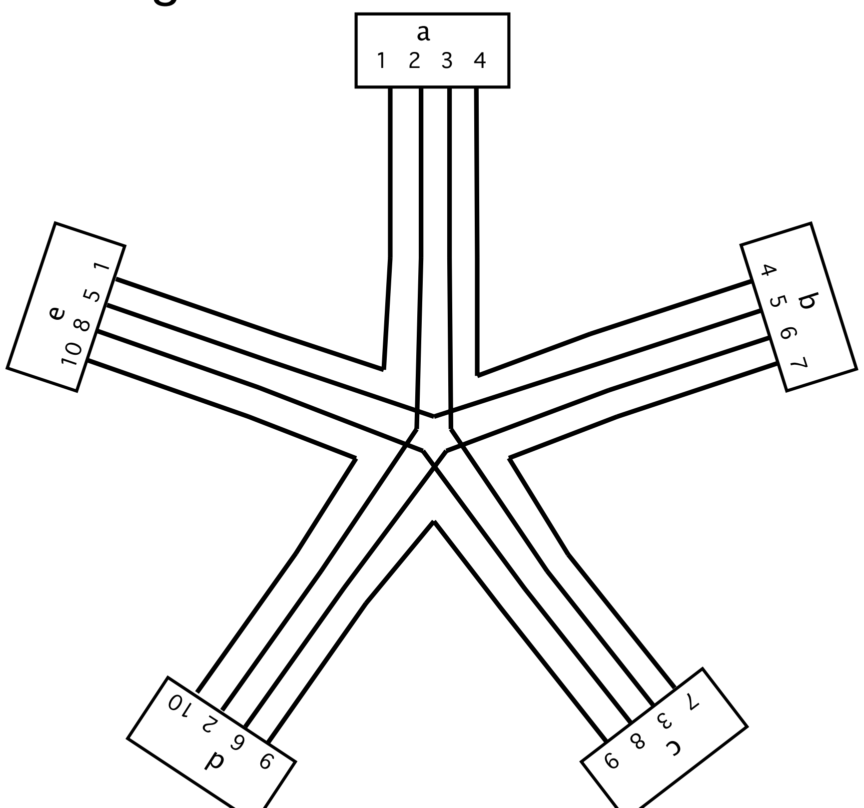

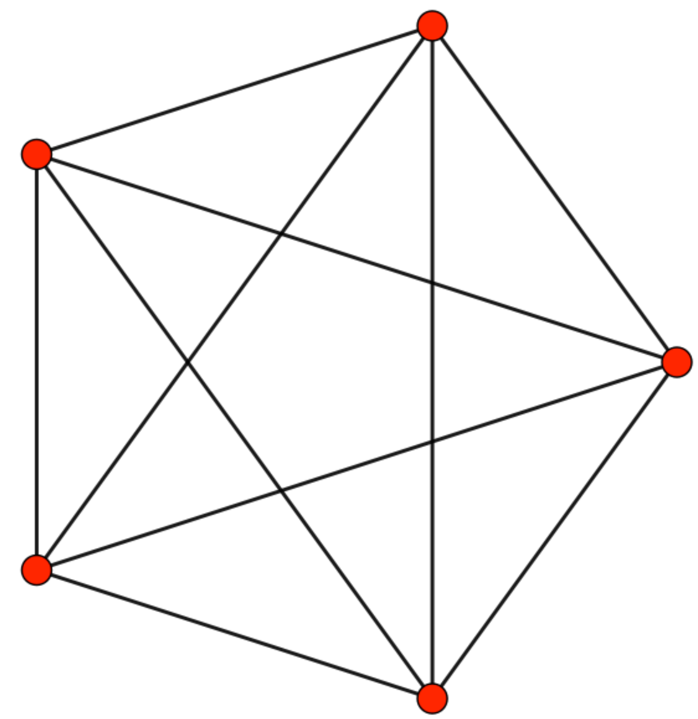

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Feynman perturbative expansion around trivial vacuum

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= stranded diagrams dual to cellular complexes of arbitrary topology

(simplicial case: simplicial complexes obtained by gluing d-simplices in arbitrary ways)

(richer combinatorics of Feynman diagrams wrt ordinary local QFT)

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Feynman amplitudes (model-dependent):

equivalently:

- spin foam models (sum-over-histories of spin networks)
  Reisenberger,Rovelli, ’00
- lattice path integrals (with group+Lie algebra variables)
  A. Baratin, DO, ‘11

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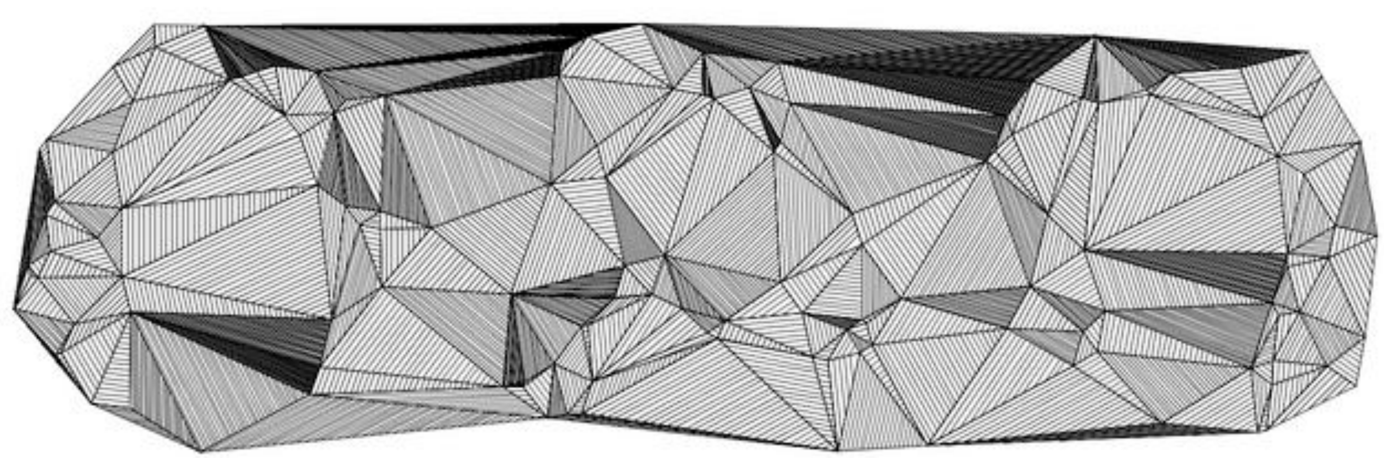

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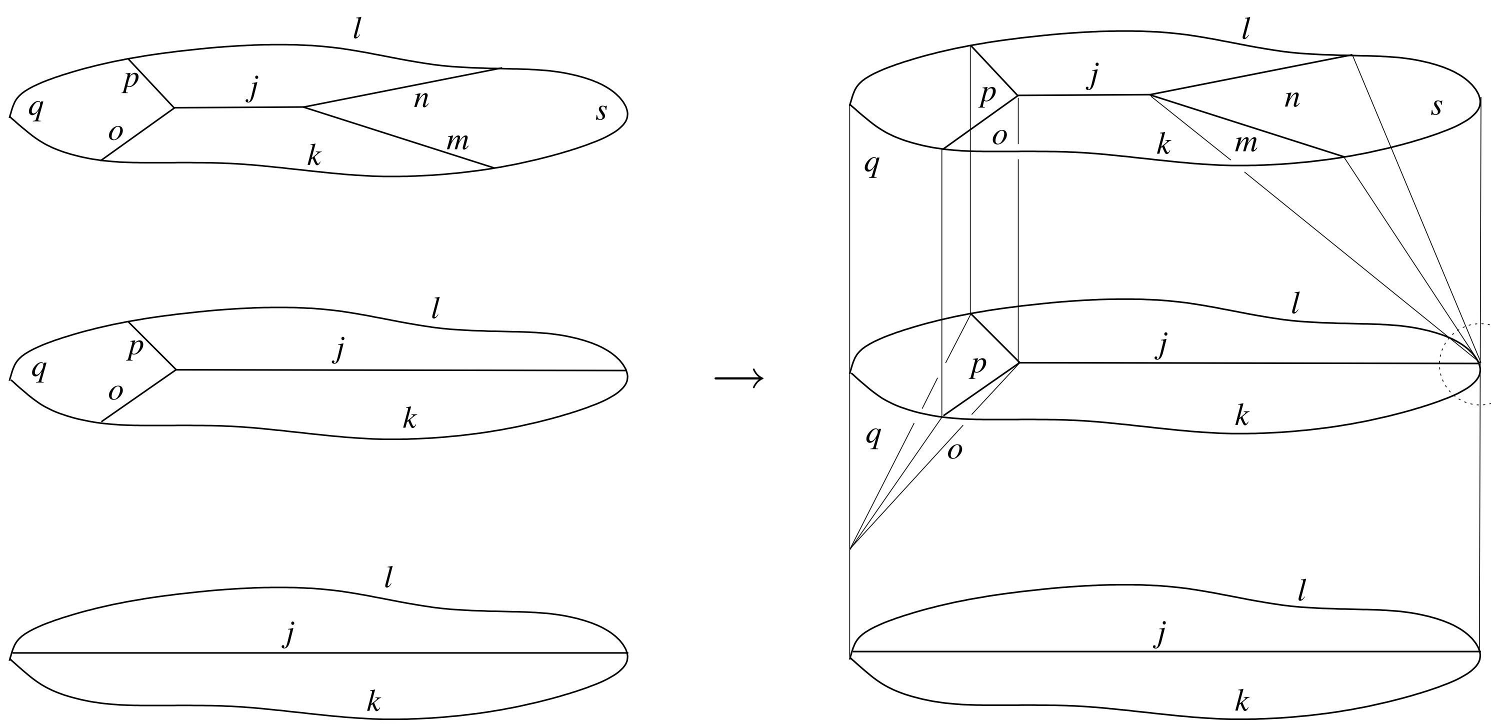

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example: d=3

dropping group/algebra data
(or restricting to finite group)

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dropping group/algebra data
(or restricting to finite group)

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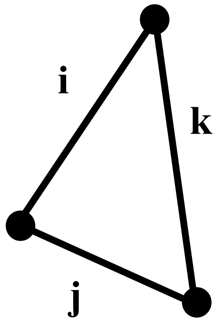

Group field theories and tensor models

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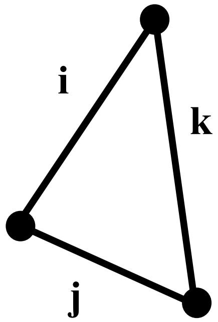

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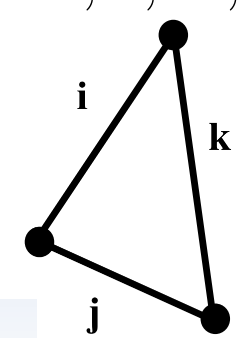

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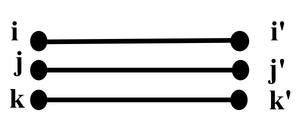

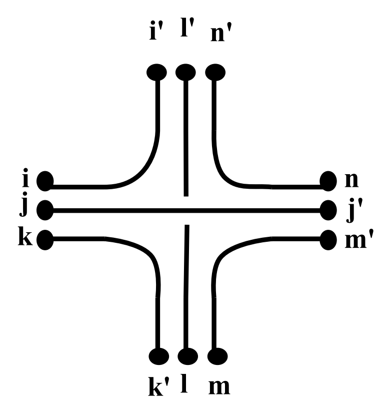

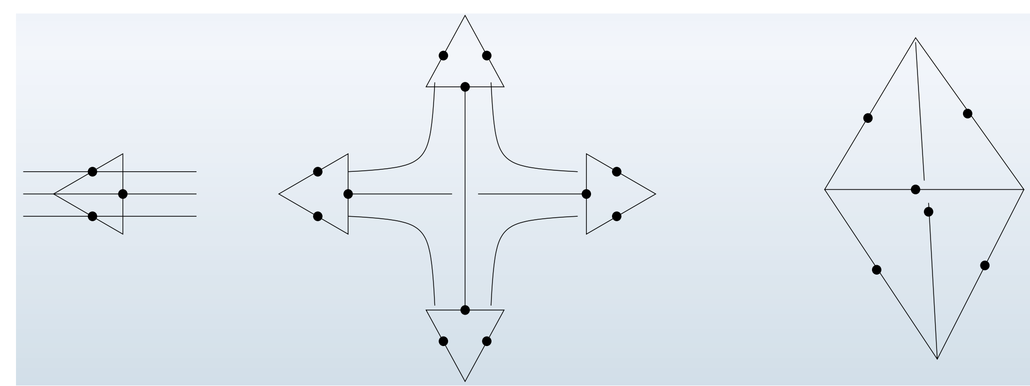

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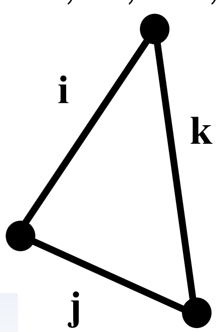

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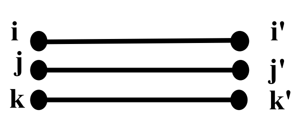

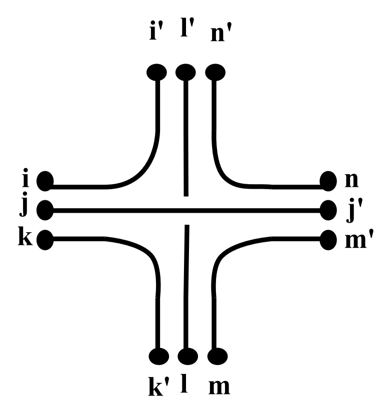

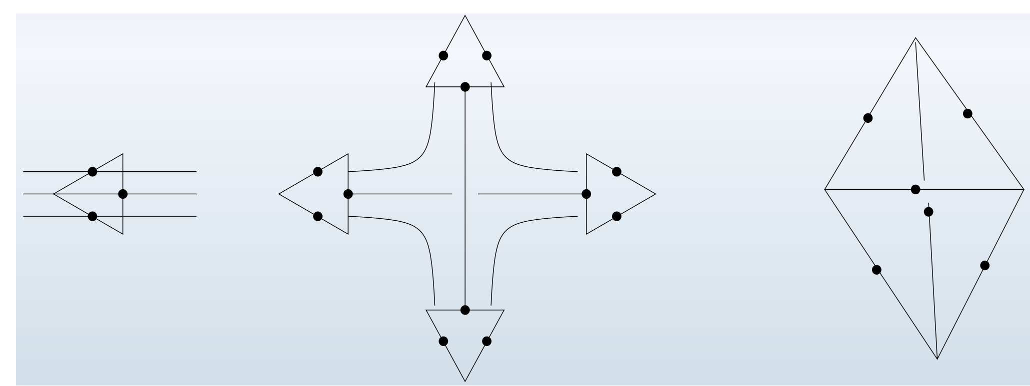

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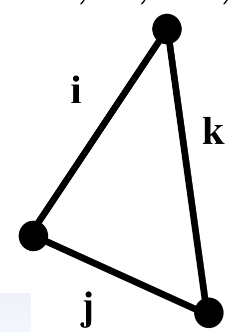

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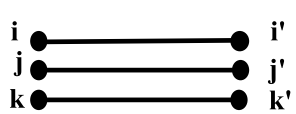

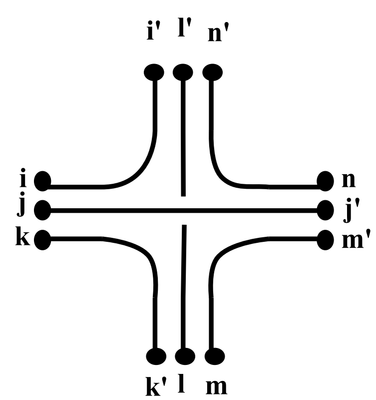

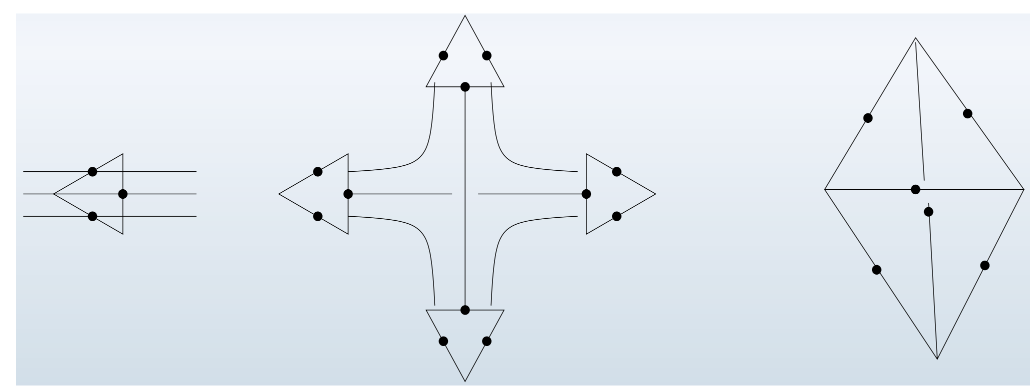

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analogous to
Dynamical
Triangulations

Group field theories

good points:

LQG spin networks as many-body systems and 2nd quantisation —-> GFT Fock space

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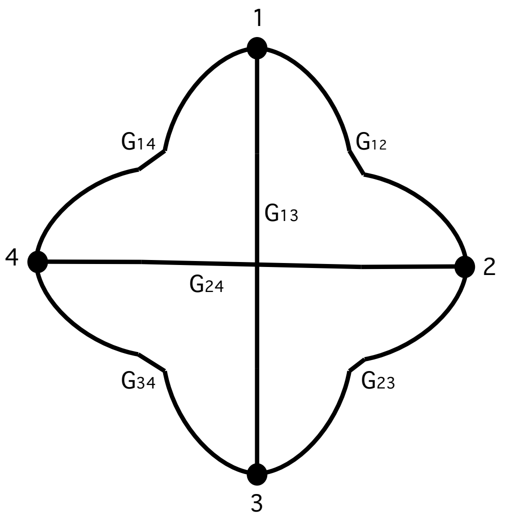

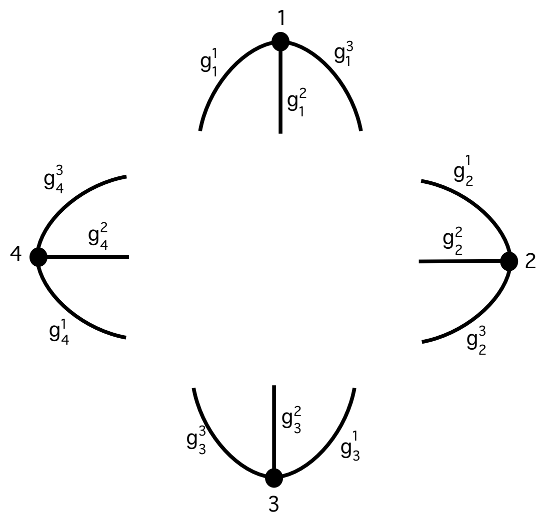

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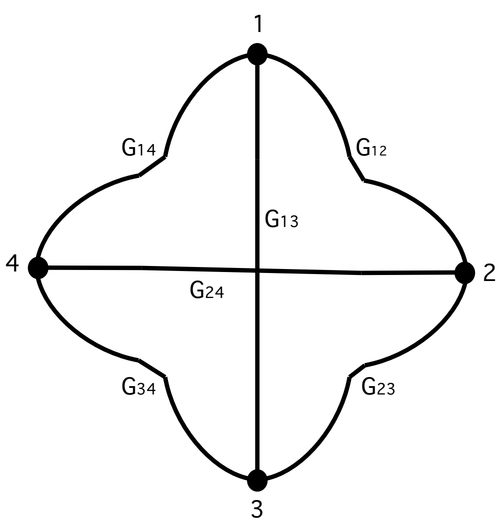

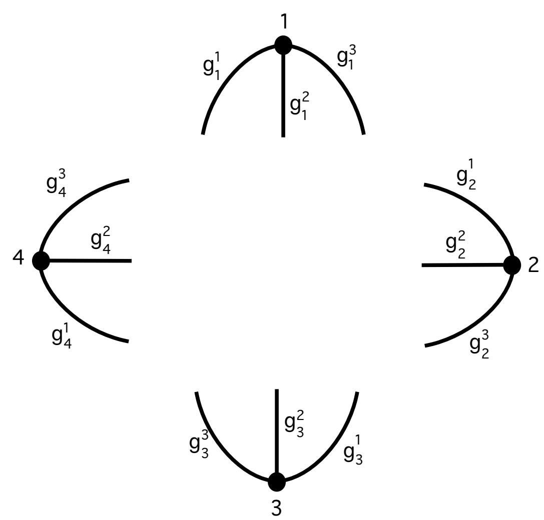

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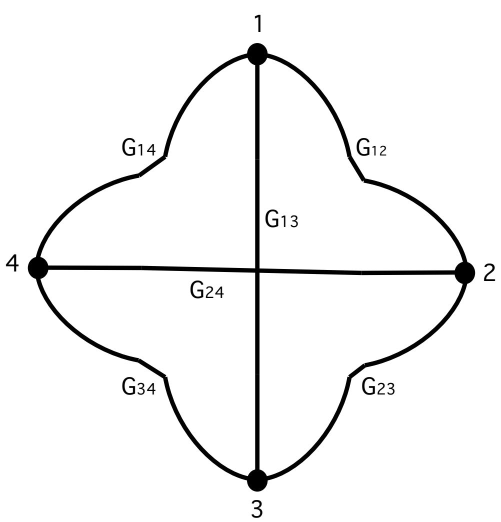

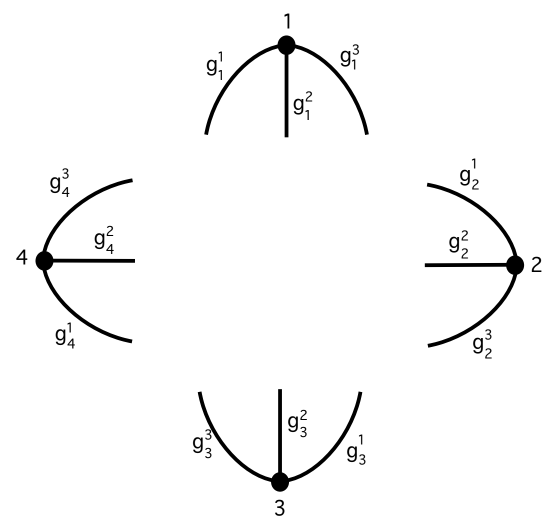

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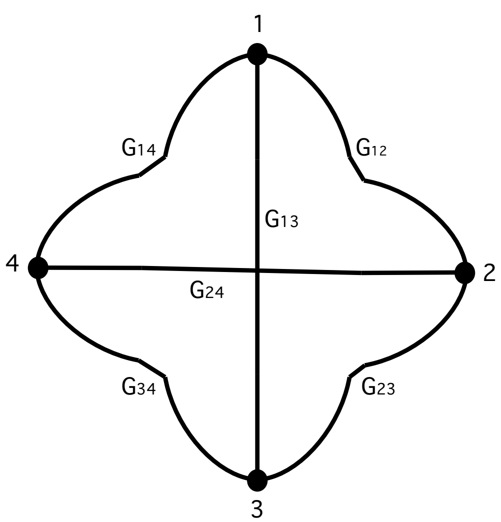

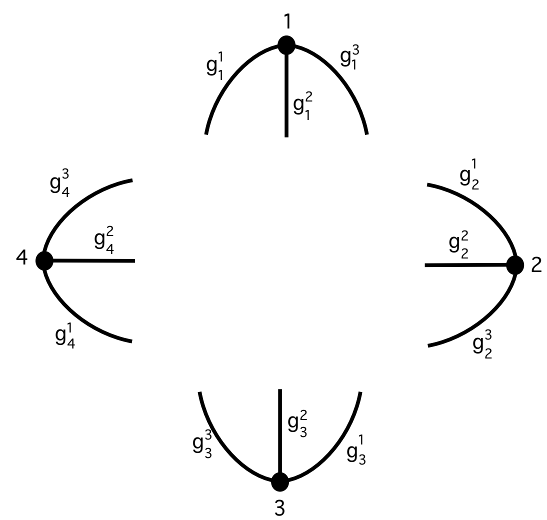

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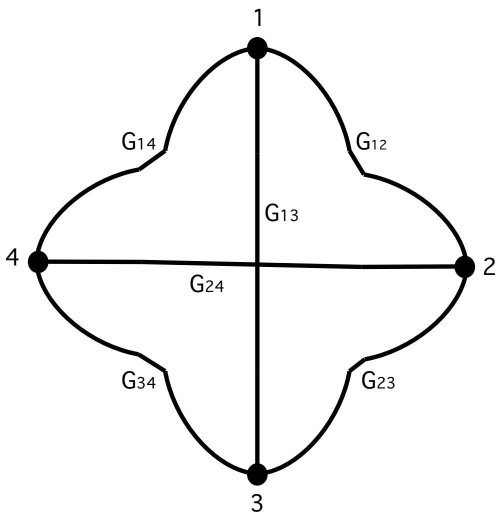

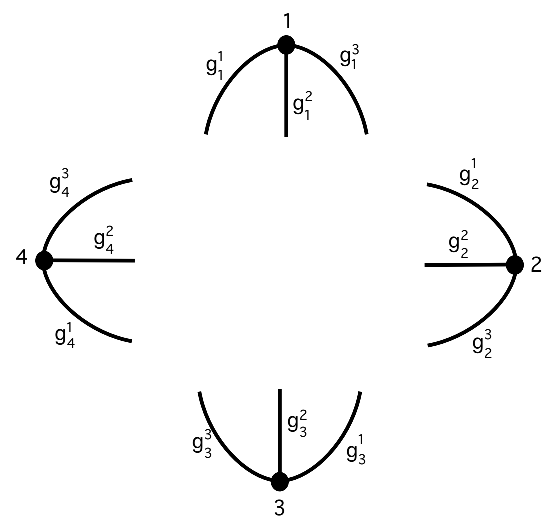

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precise prescription for combinatorial weights in sum over spin foams

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open issues:

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→ renormalisation!!!!

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so…. some people started advocating greater role for GFTs and called for taking advantage of QFT methods

This attracted the attention of Vincent ……..

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Vincent arrived on the scene……

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brought in many collaborators

quickly built up on results of others

found many interesting mathematical problems (and solutions)

opened up several new directions

Vincent's contributions to GFT renormalisation

initial work on topological group field theories

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example: d=3 $\varphi_\ell \,:\, SO(3)^3/SO(3) \,\rightarrow\, \mathbb{R}$

$$\forall h \in \mathrm{SO}(3), \qquad \varphi_\ell(hg_1, hg_2, hg_3) \,=\, \varphi_\ell(g_1, g_2, g_3)$$

\+ simplicial interaction

with only delta functions

valid for GFT definition of BF theory in any dimension

can be computed in different (equivalent) representations (group, spin, Lie algebra)

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$$\begin{aligned} S_{int}[\varphi_\ell] \quad &= \quad \lambda \int [\mathrm{d}g_i]^6 \, \varphi_1(g_1, g_2, g_3)\varphi_2(g_3, g_4, g_5)\varphi_3(g_5, g_2, g_6)\varphi_4(g_6, g_4, g_1) \\ &\quad +\lambda \int [\mathrm{d}g_i]^6 \, \overline{\varphi_4}(g_1, g_4, g_6)\overline{\varphi_3}(g_6, g_2, g_5)\overline{\varphi_2}(g_5, g_4, g_3)\overline{\varphi_1}(g_3, g_2, g_1) \end{aligned}$$

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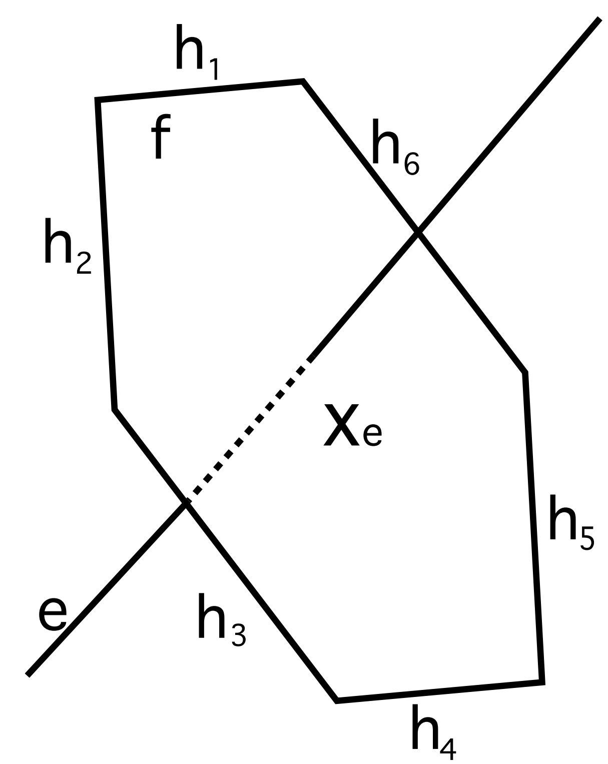

discretization of: $S(e,\omega) = \int Tr(e \wedge F(\omega))$

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lattice gauge theory formulation of 3d gravity/BF theory

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discrete 1st order path integral for 3d gravity/BF theory on simplicial complex dual to GFT Feynman diagram

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lattice gauge theory formulation of 3d gravity/BF theory

$$= \sum_{\{j_e\}} \prod_e d_{j_e} \prod_\tau \left\{ \begin{array}{ccc} j_1^\tau & j_2^\tau & j_3^\tau \\ j_4^\tau & j_5^\tau & j_6^\tau \end{array} \right\} = \int \prod_l [\mathrm{d}h_l] \prod_e [\mathrm{d}^3 x_e]\, e^{i\sum_e \mathrm{Tr}\, x_e H_e}$$

discrete 1st order path integral for 3d gravity/BF theory on simplicial complex dual to GFT Feynman diagram

spin foam formulation of 3d gravity/BF theory

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initial work on topological group field theories and models of 4d gravity

intricate divergence structure depending on combinatorics of simplicial complex

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….only warming up….no full use of QFT tools…missing ingredients in the formalism

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colouring!

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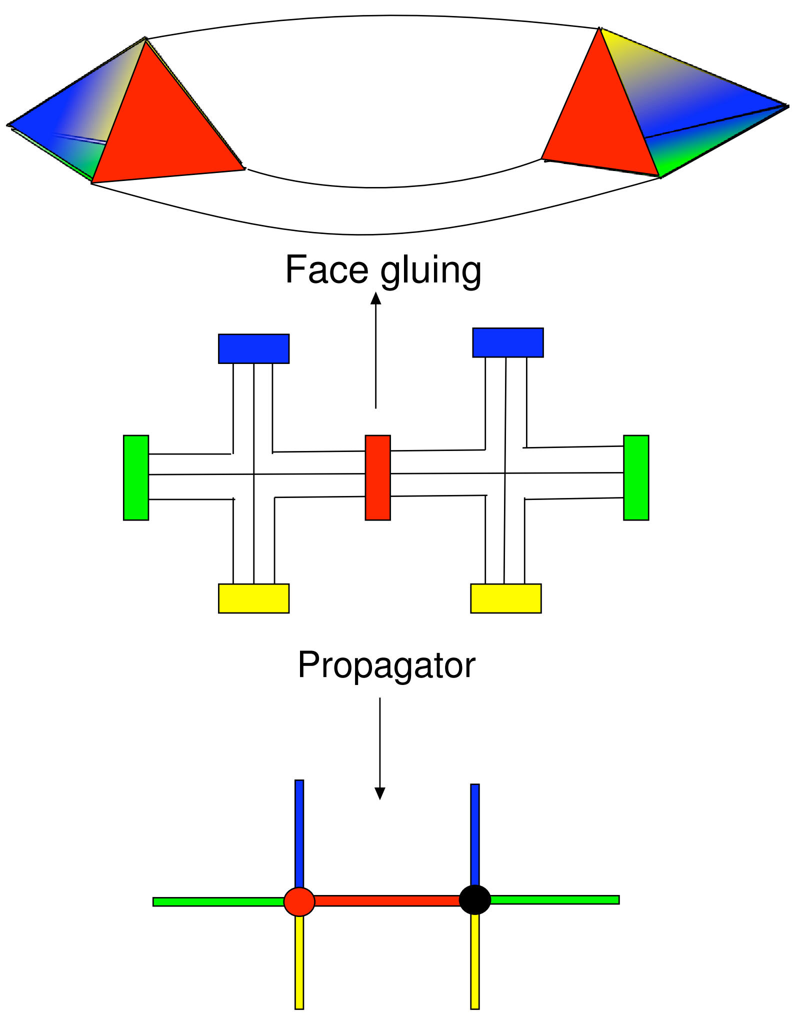
(Pezzana, Ferri, Gagliardi,...)

Every PL D-pseudomanifold M can be represented by a (D+1)-colored graph G



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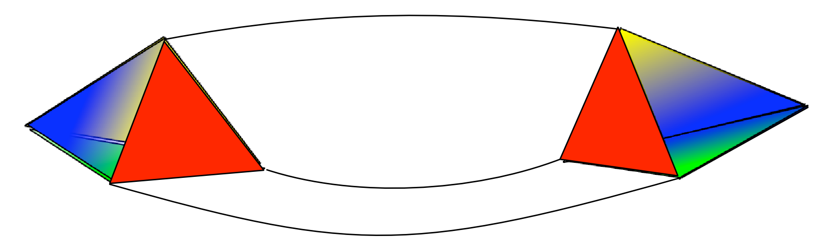
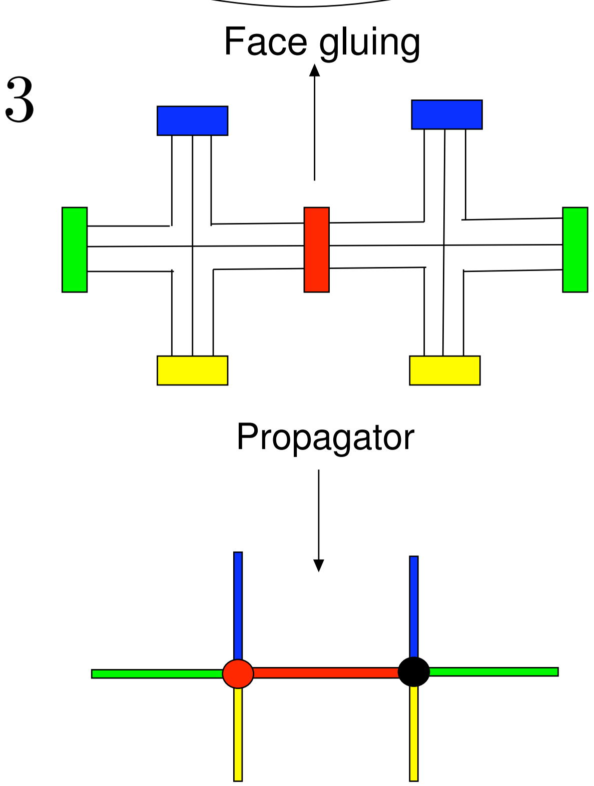


refined definition of tensor models via "colors" (......Gurau.....)

e.g. 4 complex un-symmetric tensors: $T^a_{ijk} \, : \, \mathbb{Z}_N^{\times 3} \rightarrow \mathbb{C} \qquad a = 0,1,2,3$

$$S(T) = \frac{1}{2} \sum_a \sum_{i,j,k} T^a_{ijk} \bar{T}^a_{ijk} - \frac{\lambda}{4! \sqrt{N^3}} \sum_{ijklmn} T^0_{ijk} T^1_{klm} T^2_{mjn} T^3_{nli} \quad + \quad c.c.$$



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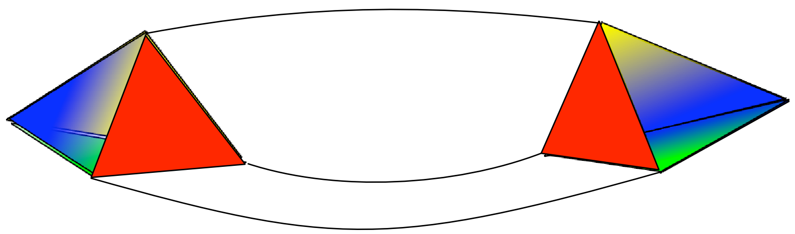

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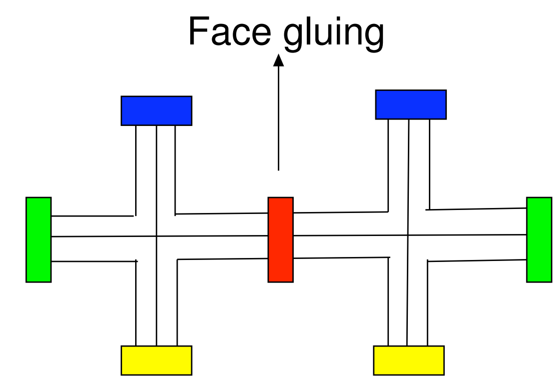

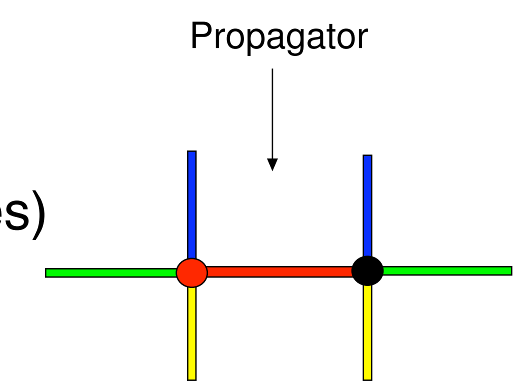

led to:

- 1/N expansion - dominance of melonic diagrams (special triangulations of spheres)
- universality of tensors
- notion of “tensorial invariance” ~ “tensorial locality”

Tensorial GFTs (key insights from tensor models)

locality principle and soft breaking of locality:

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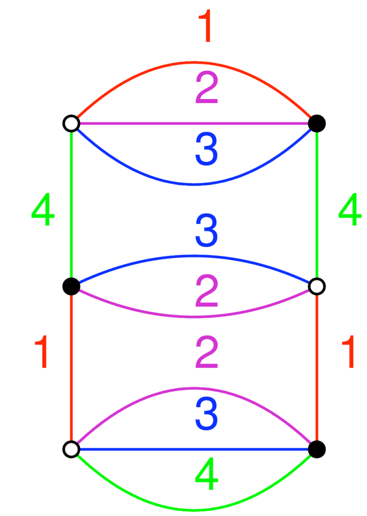
$$S(\varphi,\overline{\varphi}) = \sum_{b\in\mathcal{B}} t_b I_b(\varphi,\overline{\varphi})$$

indexed by bipartite d-colored graphs ("bubbles")
dual to d-cells with triangulated boundary



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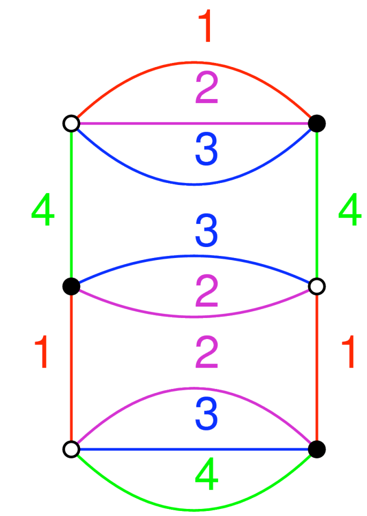

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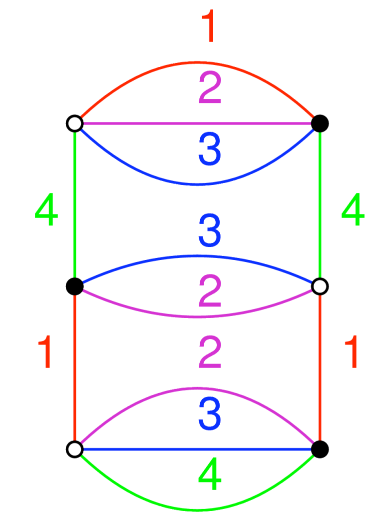

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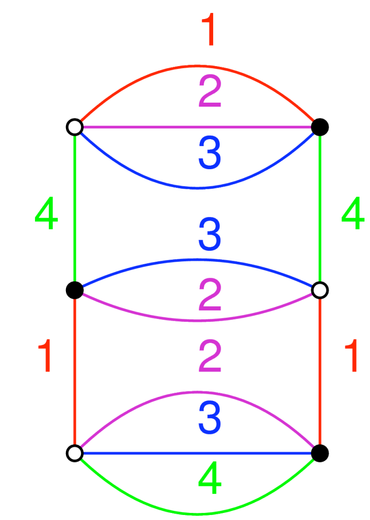

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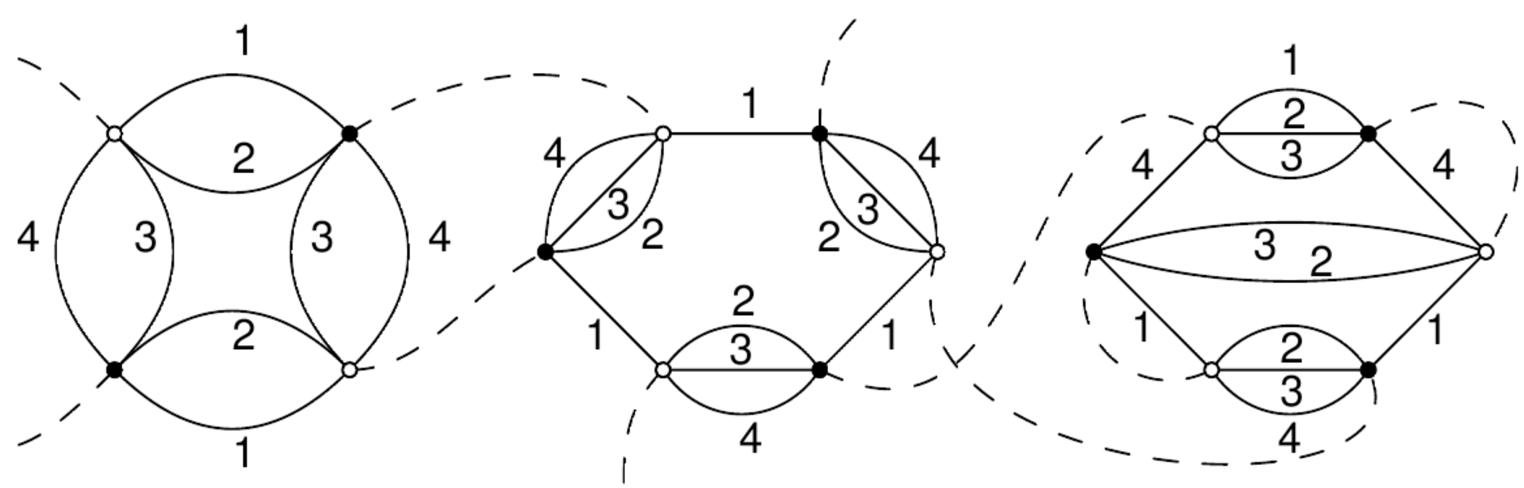

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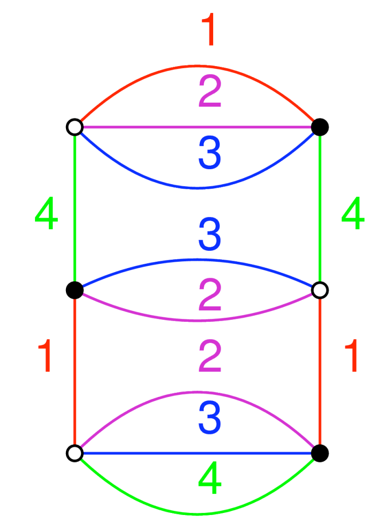

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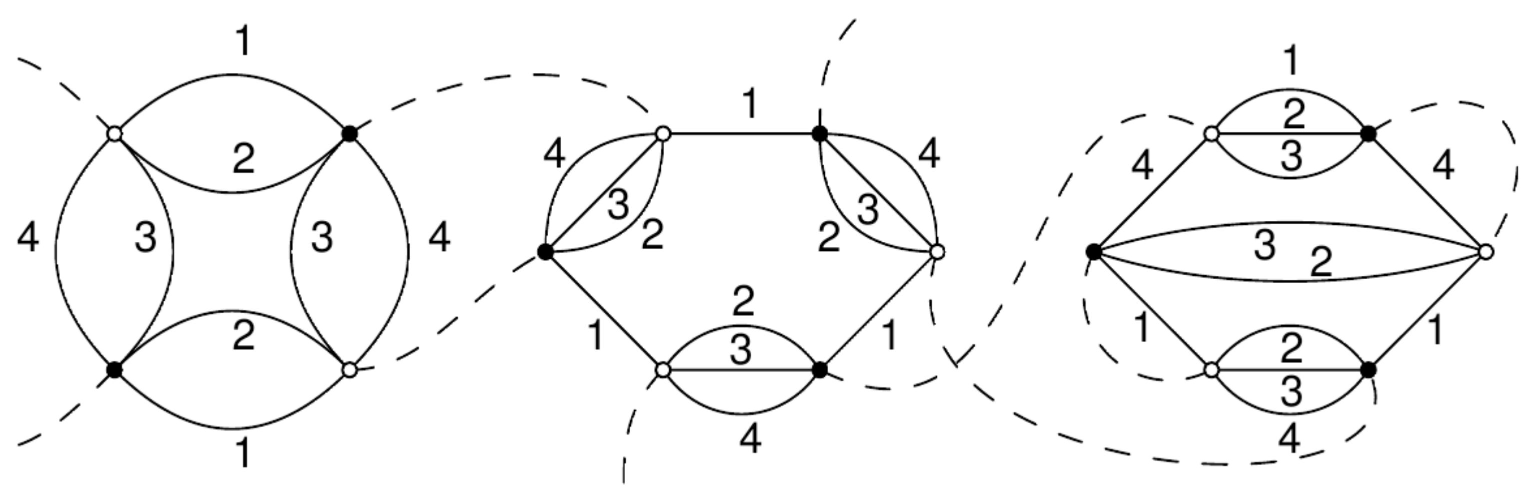
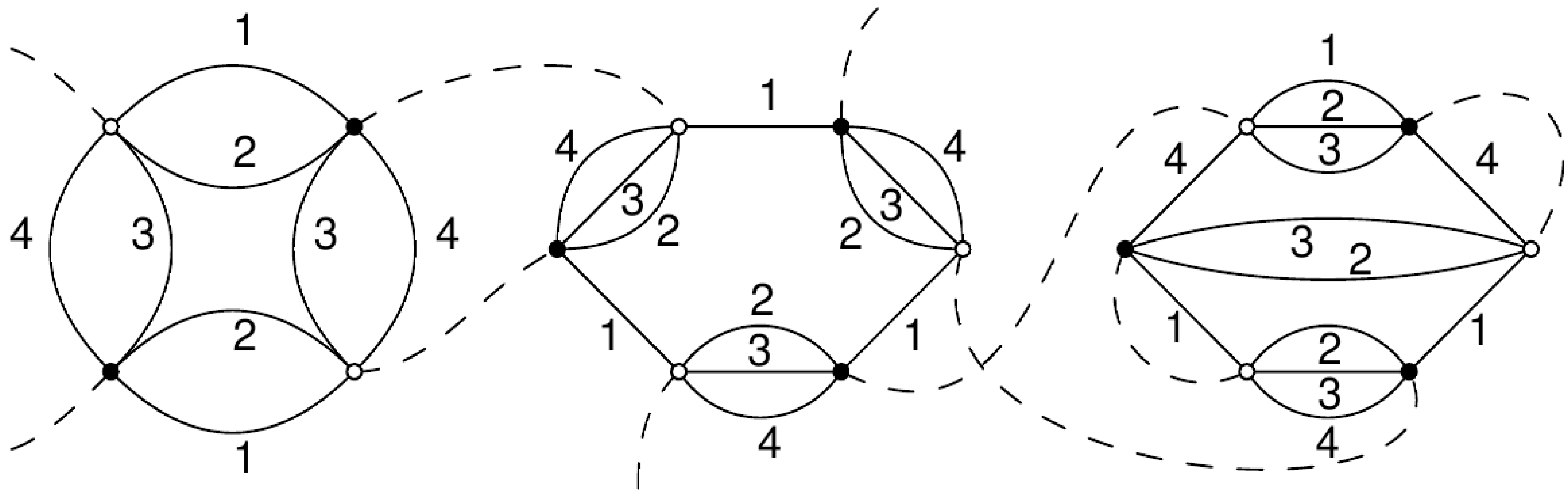


require generalization of notions of "connectedness", "contraction of high subgraphs", "locality", Wick ordering,
....
taking into account internal structure of Feynman graphs, full combinatorics of dual cellular complex, results from crystallization theory (dipole moves)

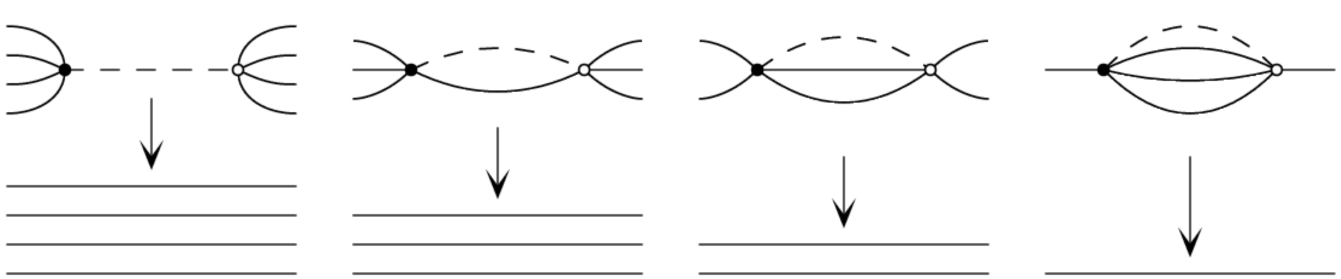
TGFT renormalization

example of Feynman diagram



- building blocks: coloured bubbles, dual to d-cells with triangulated boundary
- glued along their boundary (d-1)-simplices
- parallel transports (discrete connection) associated to dashed (color 0, propagator) lines
- faces of color i = connected set of (alternating) lines of color 0 and i

“contraction of internal line” ~ dipole contraction



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focus on tensorial field theories, independently (or before) full blown TGFTs with quantum geometric data

bring renormalisation to quantum gravity!

base theory on universality of tensors

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S. Carrozza, DO, V. Rivasseau, ‘12

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- first renormalizable non-abelian TGFT model with gauge invariance (rank-3, SU(2), 3d gravity)

S. Carrozza, DO, V. Rivasseau, ‘13

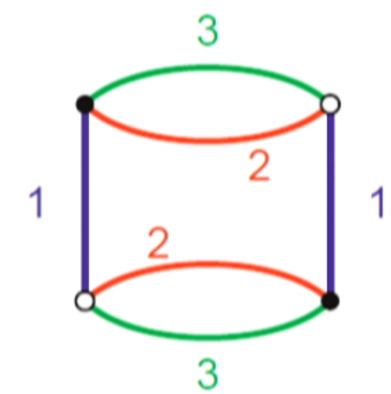
TGFT example: SU(2), d=3, with gauge invariance

Carrozza, DO, Rivasseau, '13

kinetic term = Laplacian on SU(2)^3

$$\left(m^2 - \sum_{\ell=1}^{d} \Delta_\ell\right)^{-1}$$

tensor invariant interactions, e.g.



gauge invariance: $\forall h \in G \,, \qquad \varphi(g_1, \dots, g_d) = \varphi(g_1 h, \dots, g_d h)$

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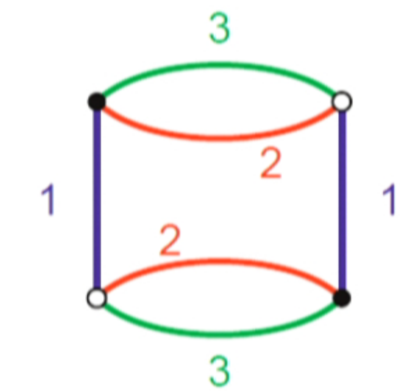

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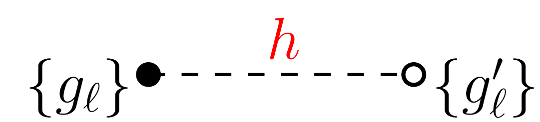

covariance (in multi-scale slicing, via heat kernel):

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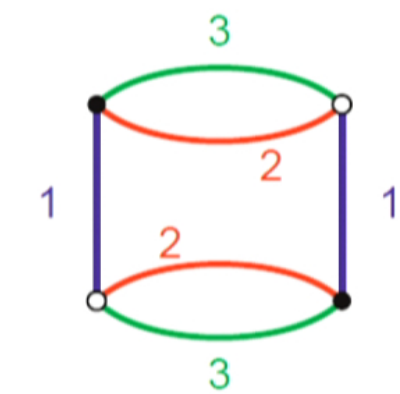

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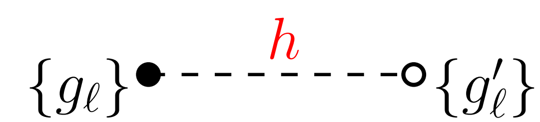

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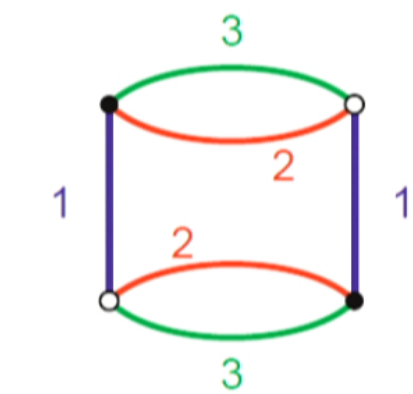

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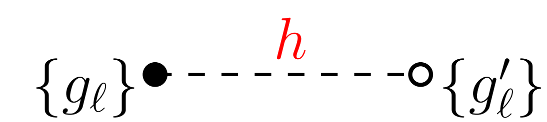

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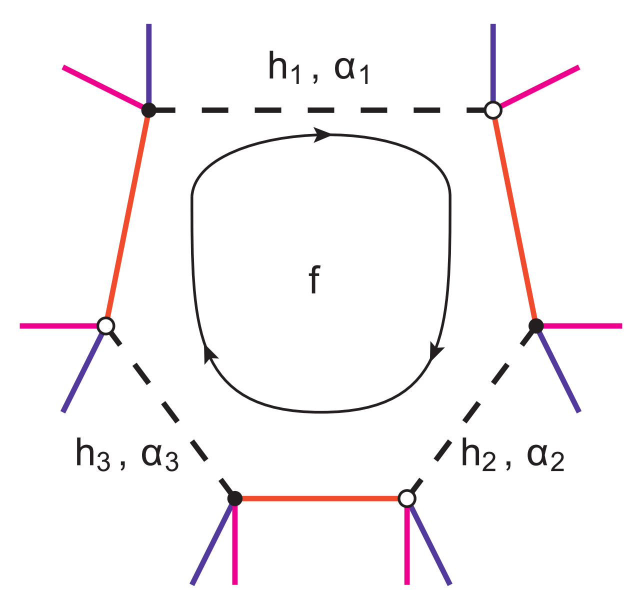
amplitudes factorise per face:



$\longleftrightarrow \qquad K_{\alpha_1 + \alpha_2 + \alpha_3}(h_1 h_2 h_3)$

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similar analysis for TGFTs on homogeneous space SU(2)/U(1) Lahoche, DO, '15

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3	4	4	$G = \mathrm{SU}(2) \times \mathrm{U}(1)$ [SC '14]
4	2	4	
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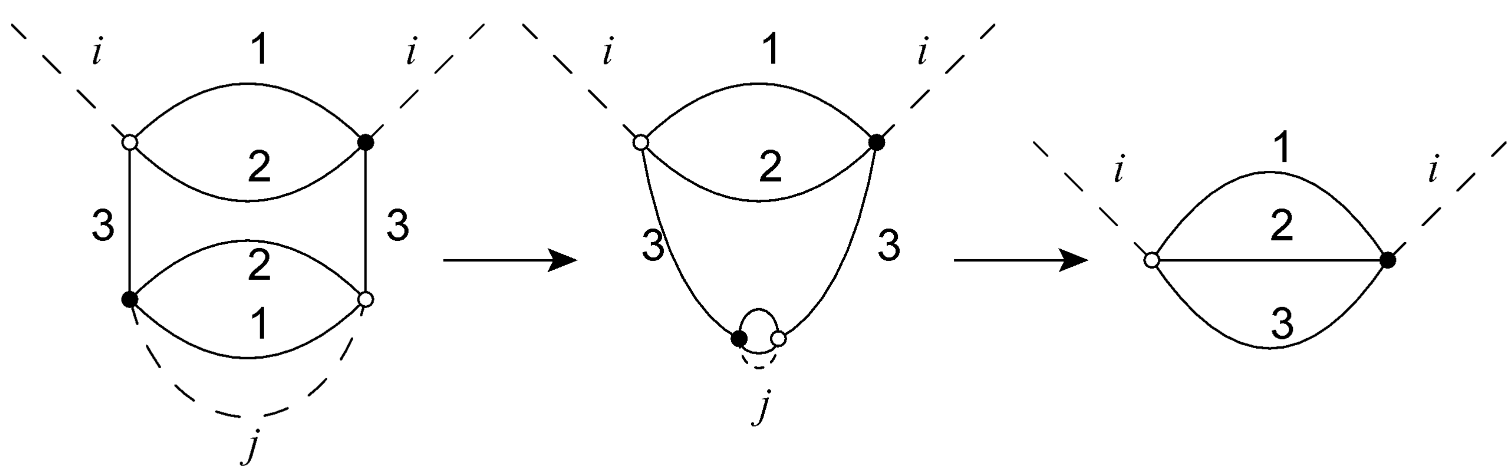

TGFT example: SU(2), d=3, with gauge invariance

Carrozza, DO, Rivasseau, '13

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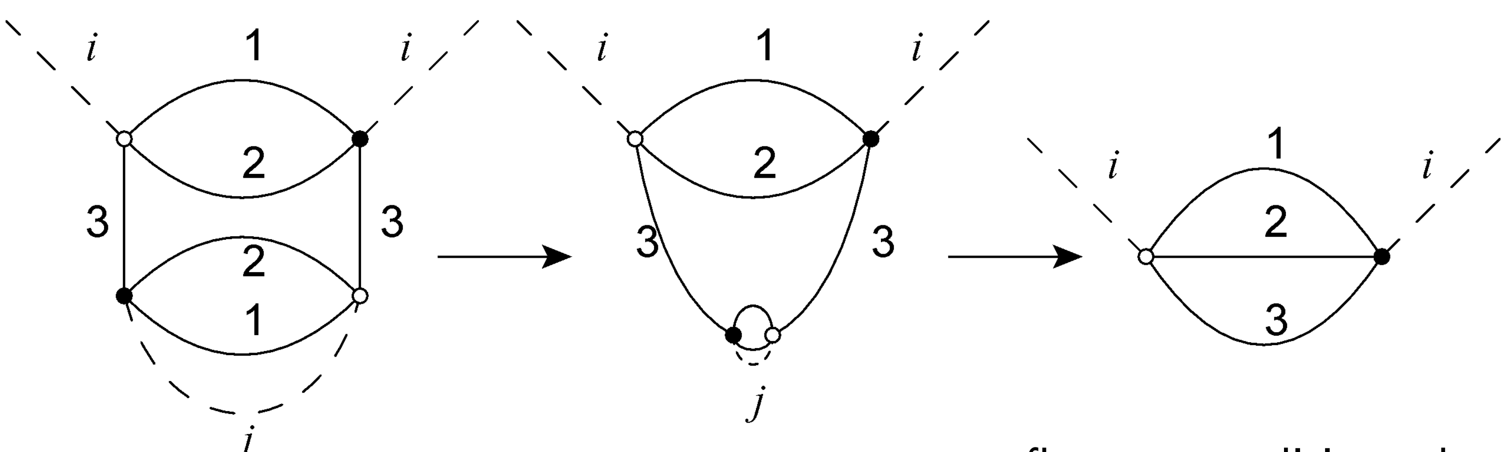

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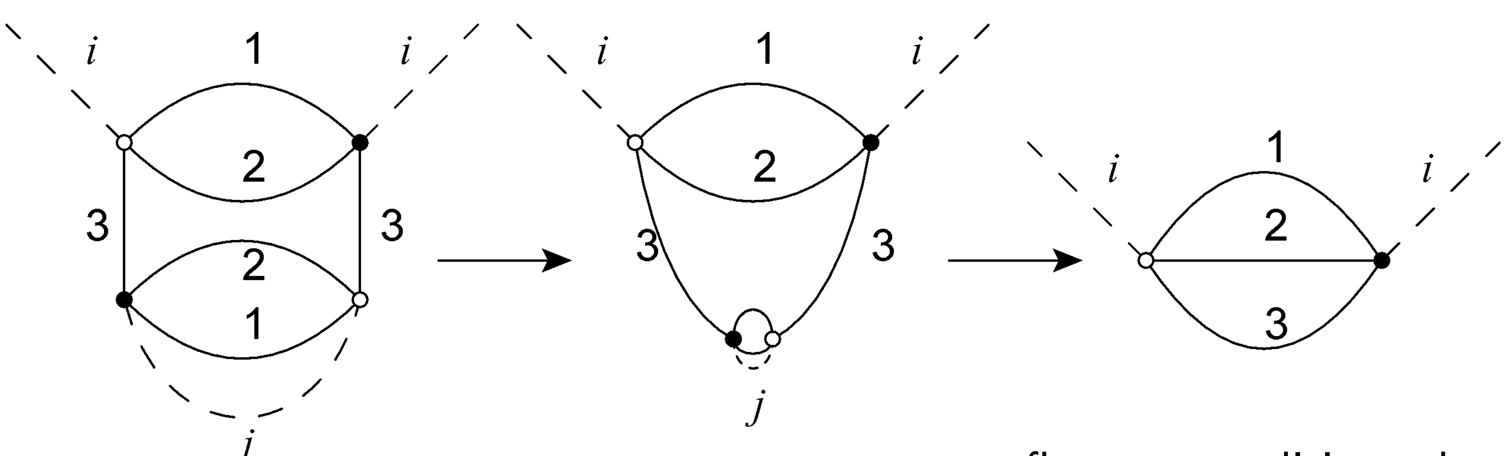

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true for models dominated by "melonic diagrams"

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- several renormalizable abelian TGFT models (different groups and dimension, with/without gauge invariance)

  J. Ben Geloun, V. Rivasseau, '11; J. Ben Geloun, D. Ousmane-Samary, '11 S. Carrozza, DO, V. Rivasseau, '12

- first renormalizable non-abelian TGFT model in 3d with gauge invariance (3d BF + laplacian)

  S. Carrozza, DO, V. Rivasseau, '13

- first renormalizable TGFT model on homogeneous space (SU(2)/U(1))^d V. Lahoche, DO, '15

- proof of asymptotic freedom for abelian TGFT models without gauge invariance

  J. Ben Geloun, D. Ousmane-Samary, '11; J. Ben Geloun, '12

- study of asymptotic freedom/safety for non-abelian TGFT models with gauge invariance

  S. Carrozza, '14

- 4th order interactions: generic asymptotic freedom (strong wave function renorm.); higher orders: more subtle

- ……………

Vincent’s contributions to GFT renormalisation

much more along the Tensor Track!

V. Rivasseau, ‘14

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- Functional RG approach to GFTs -

Krajewski,Toriumi, ’14; Benedetti, Ben Geloun, DO, ’14; Ben Geloun, Martini, DO, ’15; Benedetti, Lahoche, ’15; …..

after Vincent arrived on the scene……

a very wet landscape…..not easily recognisable….

….. but one that has become very fertile and rich!

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- many new tools
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the whole GFT field is thriving like never before (not only renormalisation or statistical aspects, of course)!

the multi-scale tsunami “Vincent” is constituted of some strange, beneficial, energetic fluid!

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friendly and very human tsunami:
constant support and encouragement

Happy birthday, Vincent!

and, Thanks!