From non-commutative quantum field theory to tensor models and Combinatorial Physics - my scientific interactions with Vincent Rivasseau

ADRIAN TANASĂ

univ. Bordeaux & I.U.F.

IHP, 26th of November 2015

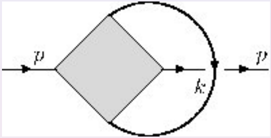
(Scalar) QFT on the non-commutative Moyal space

non-commutative action:

$$S^\star[\Phi(x)] = \int d^4x \, \left(\frac{1}{2} \sum_{\mu=1}^{4} (\frac{\partial \Phi(x)}{\partial x_\mu})^{\star 2} + \frac{1}{2} m^2 \Phi^{\star 2}(x) + \frac{\lambda}{4!} \Phi^{\star 4}(x) \right) .$$

$\star$ - the non-commutative Moyal product

Feynman integral computations - renormalizability



$$\int d^4k \frac{e^{\imath k_\mu \Theta^{\mu\nu} p_\nu}}{k^2+m^2} = \sqrt{\frac{m}{(\Theta \cdot p)^2}} K_1 \left(\sqrt{m^2 (\Theta \cdot p)^2} \right) \to_{|p| \to 0} \frac{1}{\theta^2 p^2}$$

$\Rightarrow$ non-renormalisable model

same type of behaviour of the 2−point function at any order in perturbation theory

J. Magnen, V. R. and A. T., *Europhys. Lett.* ('09), arXiv:0807.4093[hep-th]

Solutions - modifications of the quadratic part of the action:
2 types of solutions

1. additional harmonic term - the Grosse-Wulkenhaar model
   parametric representation of a Grosse-Wulkenhaar-like model,
   (VR and AT, *Commun. Math. Phys.*, 2008)
   Mellin representation (R. Gurău, A. Malbouisson, VR and AT , *Lett. Math. Phys.*, '07)

   not invariant under translations

   see talk of Garald Grosse (this afternoon)

A 2nd solution - translation-invariance

(R. Gurău, J. Magnen, VR and AT, *Commun. Math. Phys.* ('09)

complete propagator

$$C(p,m,\theta) = \frac{1}{p^2 + m^2 + a\frac{1}{\theta^2 p^2}}$$

modification of the quadratic part of the action:

$$\Delta S[\Phi] = \int d^4p \, \frac{1}{2} \, \frac{a}{\theta^2 p^2} \, \Phi^2$$

$\Longrightarrow$ the model becomes renormalizable, at any order in perturbation theory
(multi-scale analysis proof)

parametric representation - relation with the Bollobás-Riordan polynomial

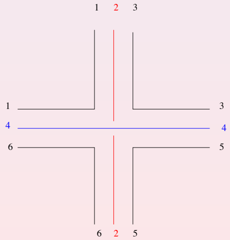
T. Krajewski, V.R, A. T. and Z. Wang *J. Noncomm. Geom.* (2010) arXiv:0811.0186 [math-ph]

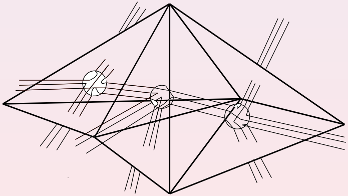
From matrices to tensors

Tensor models were introduced already in the 90's - replicate in dimensions higher than 2 the success of **random matrix models**:

natural generalization of matrix models

matrix $\rightarrow$ rank three tensor



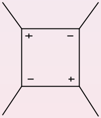


see the talks of Daniele Oriti and Stéphane Dartois (Friday) - colored tensor models

A (Moyal) QFT-inspired simplification of tensor models

highly non-trivial combinatorics and topology
$\rightarrow$ a QFT simplification of these models - the multi-orientable model

A. T., J. Phys. **A** (2012)



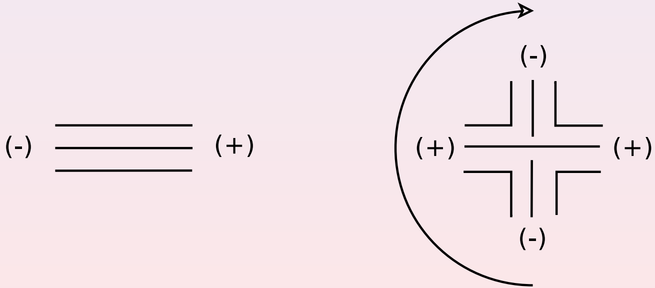
edge going from a $+$ to a $-$ corner

non-commutative QFT inspired idea

The multi-orientable tensor model

$$S[\phi] = S_0[\phi] + S_{int}[\phi],$$

$$S_0[\phi] = \frac{1}{2}\sum_{i,j,k=1}^{N} \bar{\phi}_{ijk}\phi_{ijk}, \quad S_{int}[\phi] = \frac{\lambda}{4}\sum_{i,j,k,i',j',k'=1}^{N} \phi_{ijk}\bar{\phi}_{kj'i'}\phi_{k'ji'}\bar{\phi}_{k'j'i}.$$



large N expansion of the multi-orientable tensor model

S. Dartois, V. R. and A. T., *AHP* (2014) arXiv:1301.1535[hep-th]

My scientific interaction with Vincent (2006-today)

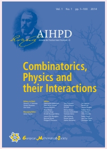
- 10 joint papers (journals of Mathematical Physics, Mathematics or Combinatorics)
- joint interactions with others scientists: J. Magnen and R. Gurau, T. Krajewski, S. Dartois, Z. Wang, R. Avohou, P. Vitale, A. Malbouisson
- 1 joint PhD student - S. Dartois (see talk on Friday)
- 1 joint workshop organization: Quantum Gravity in Paris 2015
- 2 CNRS PEPS grants and 1 ANR grant

Creation of an international journal

Annales Institut Heni Poincaré **D** - Combinatorics, Physics and their Interactions

dedicated to the growing interface between Combinatorics and Physics

contrat signed in July 2013 by *C. Villani (IHP) and T. Hintermann (EMS-Publishing House)*



Founding editors and current journal leadership team:
G. Duchamp, VR, AS and AT

Thank you for your attention

and ...

Joyeuse anniversaire Vincent !

La mulți ani Vincent!

Espace de Moyal

L'algèbre de Moyal est l'espace vectoriel des fonctions $\mathcal{S}(\mathbb{R}^{\mathcal{D}})$ équipé du *produit de Moyal*

$$(f \star g)(x) = \int \frac{d^D k}{(2\pi)^D} d^D y \, f(x + \frac{1}{2}\Theta \cdot k) g(x+y) e^{ik\cdot y}.$$

$\star$ - produit de Moyal

$D = 4$ (dimension de l'espace-temps)

$$\Theta = \begin{pmatrix} \Theta_2 & 0 \\ 0 & \Theta_2 \end{pmatrix}, \;\; \Theta_2 = \begin{pmatrix} 0 & -\theta \\ \theta & 0 \end{pmatrix}.$$

θ - paramètre de noncommutativité

$$\lim_{\theta \to 0} (f \star g)(x) = f(x) g(x), \quad \forall f, g$$

Implications de l'utilisation du produit de Moyal

partie quadratique de l'action:

$$\int d^4x\,(\Phi \star \Phi)(x) = \int d^4x\,\Phi(x)\,\Phi(x), \quad \forall \Phi$$

le terme d'interaction (dans l'espace des positions)

$$\int d^Dx \Phi^{\star\,4}(x) \propto \int \prod_{i=1}^{4} d^D x_i \Phi(x_i) \delta(x_1 - x_2 + x_3 - x_4)$$

$$e^{2i \sum_{1\le i<j\le 4} (-1)^{i+j+1} x_i \Theta^{-1} x_j}$$

$\hookrightarrow$ non-localité

$\rightarrow$ distinction claire entre les graphes planaires et non-planaires